Mathematical Logic

Part Two

Recap from Last Time

Take out a sheet of paper!

What's the truth table for the → connective?

What's the negation of $p \to q$?

Some muggle is intelligent.

$$\exists m.\ (Muggle(m) \land Intelligent(m))$$

∃ is the ***existential quantifier*** and says "for some choice of m, the following is true."

Some Technical Details

Variables and Quantifiers

- Each quantifier has two parts:
  - the variable that is introduced, and
  - the statement that's being quantified.
- The variable introduced is scoped just to the statement being quantified.

$$(\exists x.\ Loves(You, x)) \land (\exists y.\ Loves(y, You))$$

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$$(\exists x.\ \mathit{Loves}(\mathit{You},\ x)) \land (\exists y.\ \mathit{Loves}(y,\ \mathit{You}))$$

The variable x just lives here.

The variable y just lives here.

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$$(\exists x.\ \mathit{Loves}(\mathit{You},\ x)) \land (\exists x.\ \mathit{Loves}(x,\ \mathit{You}))$$

The variable x just lives here.

A different variable, also named x, just lives here.

Operator Precedence (Again)

- When writing out a formula in first-order logic, quantifiers have precedence just below ¬.
- The statement

$$\exists x.\ P(x) \land R(x) \land Q(x)$$

is parsed like this:

$$(\exists x.\ P(x)) \ \land\ (R(x) \land Q(x))$$

- This is syntactically invalid because the variable x is out of scope in the back half of the formula.
- To ensure that x is properly quantified, explicitly put parentheses around the region you want to quantify:

$$\exists x.\ (P(x) \land R(x) \land Q(x))$$

“For any natural number n,
n is even if and only if n^2 is even”

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$$\forall n.\ (n \in \mathbb{N} \to (Even(n) \leftrightarrow Even(n^2)))$$

"For any natural number n,
n is even if and only if n^2 is even"

$$\forall n.\ (n \in \mathbb{N} \rightarrow (Even(n) \leftrightarrow Even(n^2)))$$

∀ is the ***universal quantifier*** and says "for any choice of n, the following is true."

The Universal Quantifier

- A statement of the form

$$\forall x.\ \textit{some-formula}$$

  is true if, for every choice of x, the statement ***some-formula*** is true when x is plugged into it.

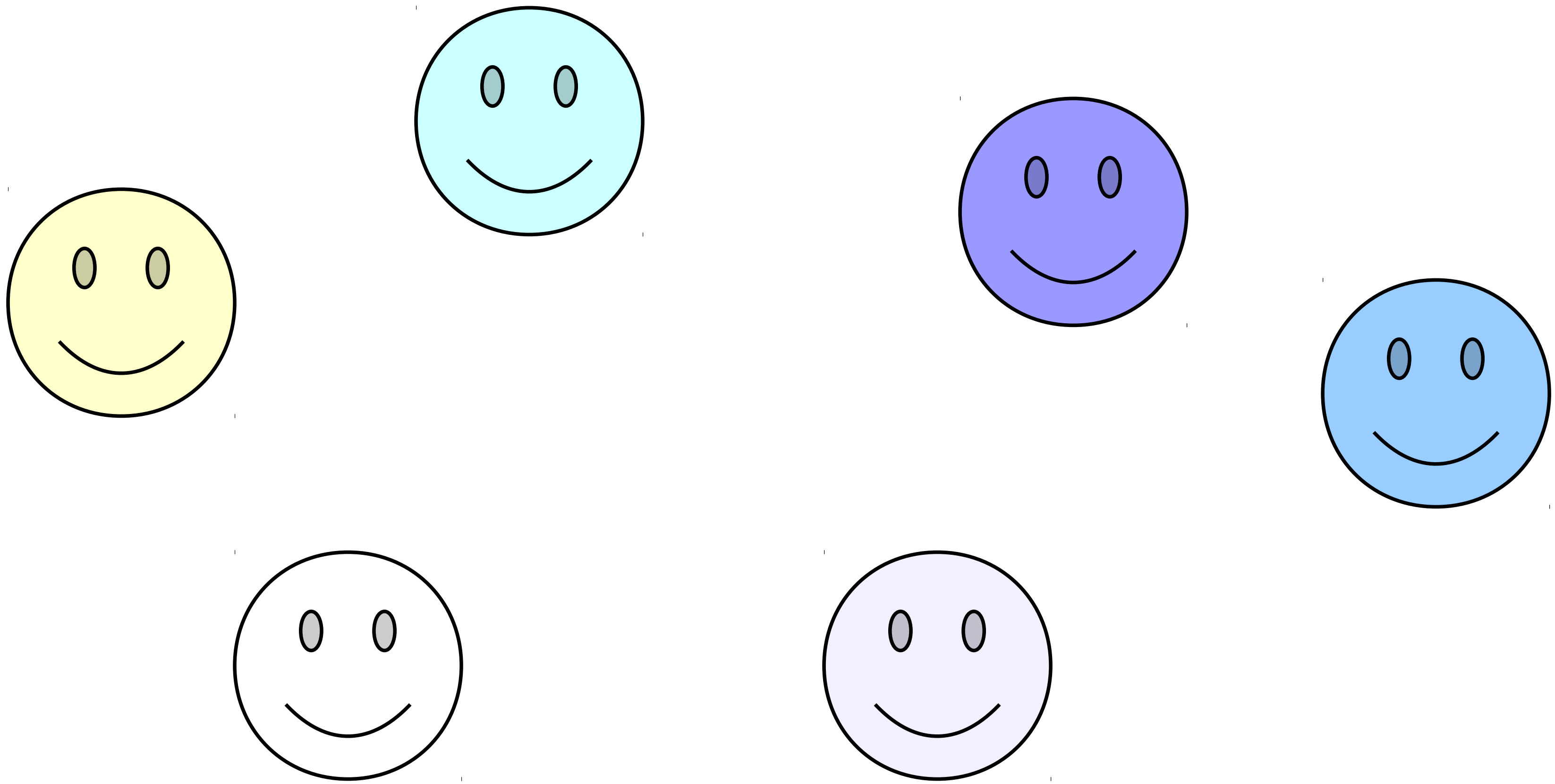
- Examples:

$$\forall p.\ (Puppy(p) \to Cute(p))$$

$$\forall a.\ (EatsPlants(a) \lor EatsAnimals(a))$$

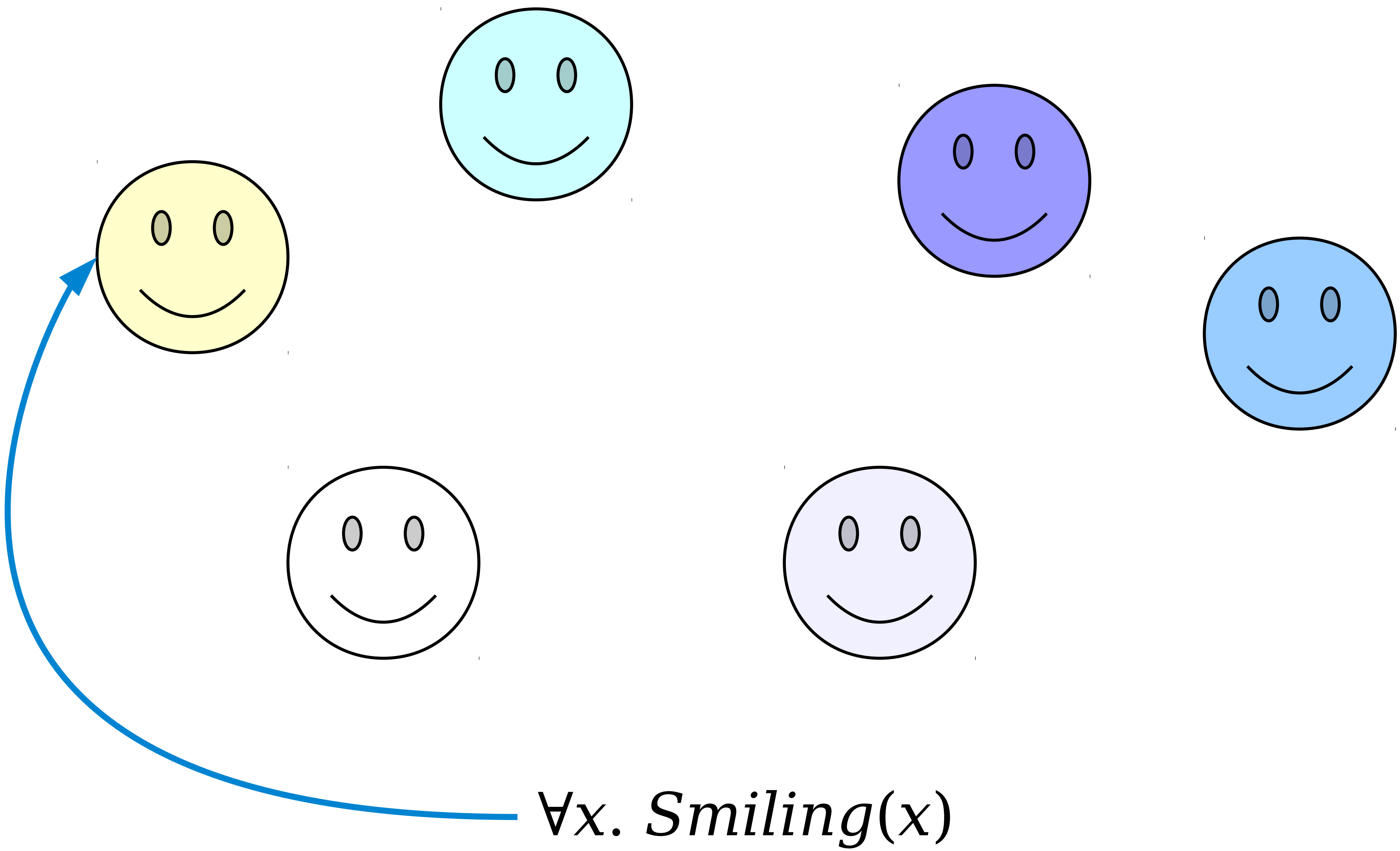
$$Tallest(SultanKösen) \to \forall x.\ (SultanKösen \neq x \to ShorterThan(x, SultanKösen))$$

The Universal Quantifier



$$\forall x.\ Smiling(x)$$

The Universal Quantifier



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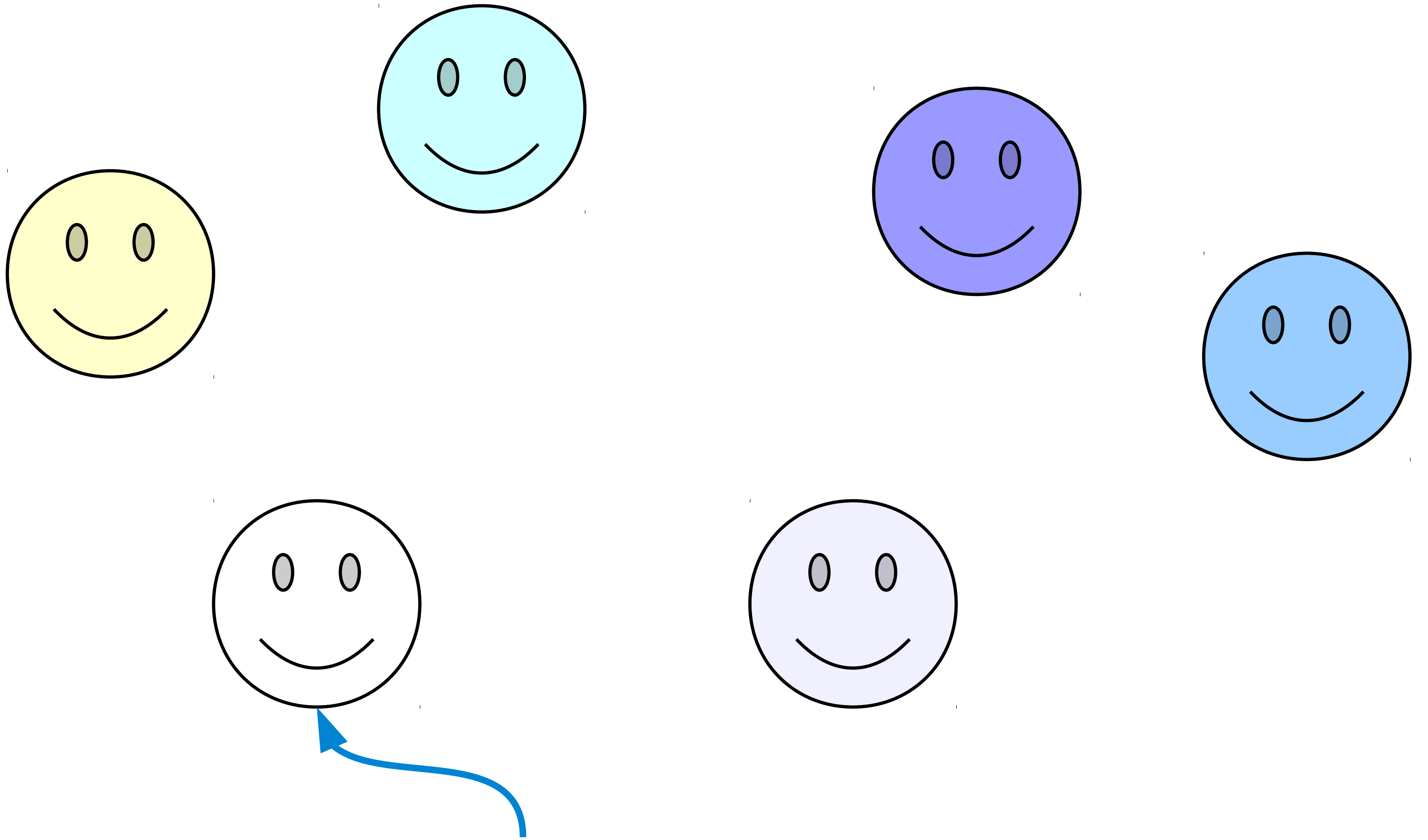

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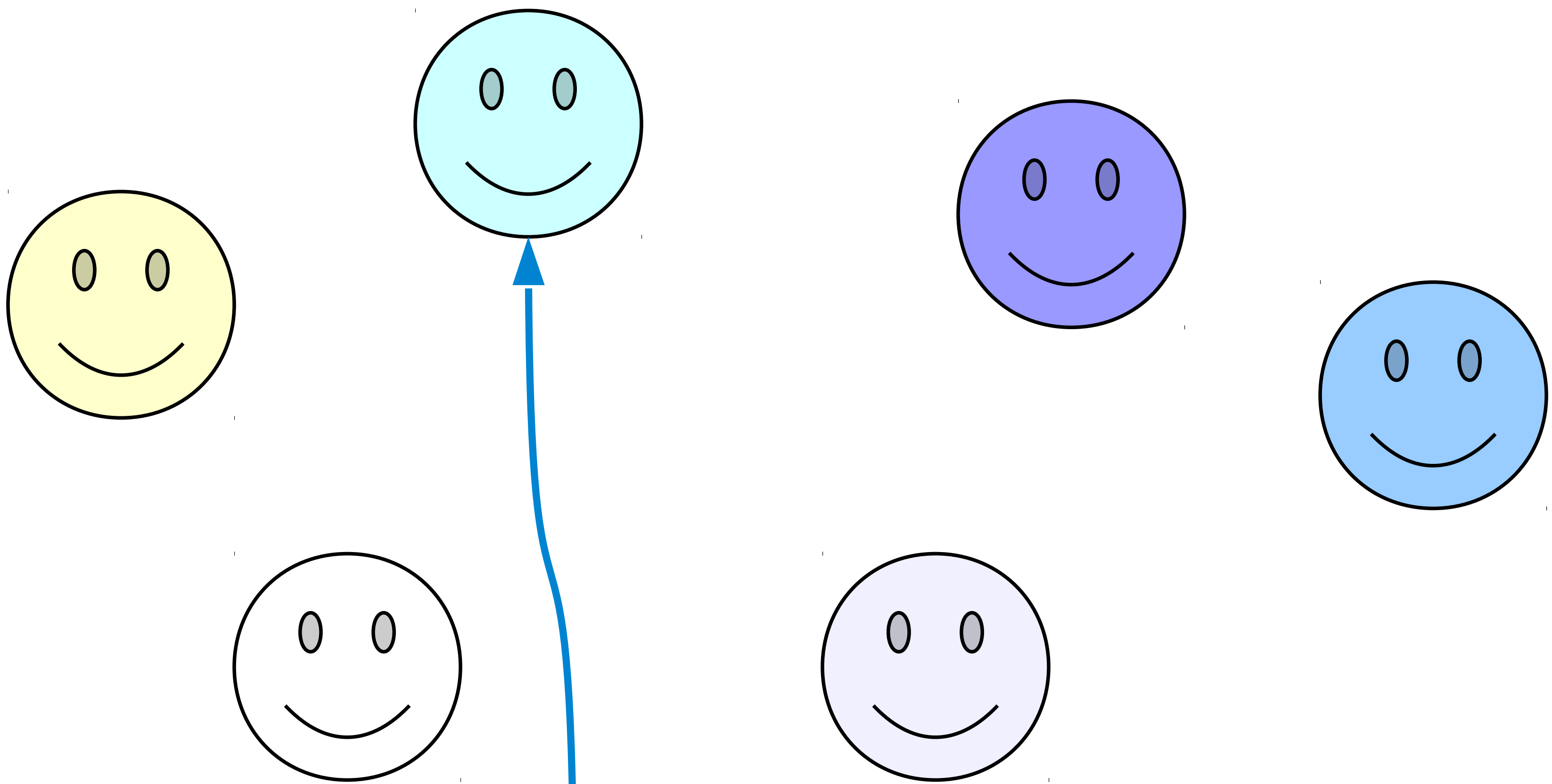

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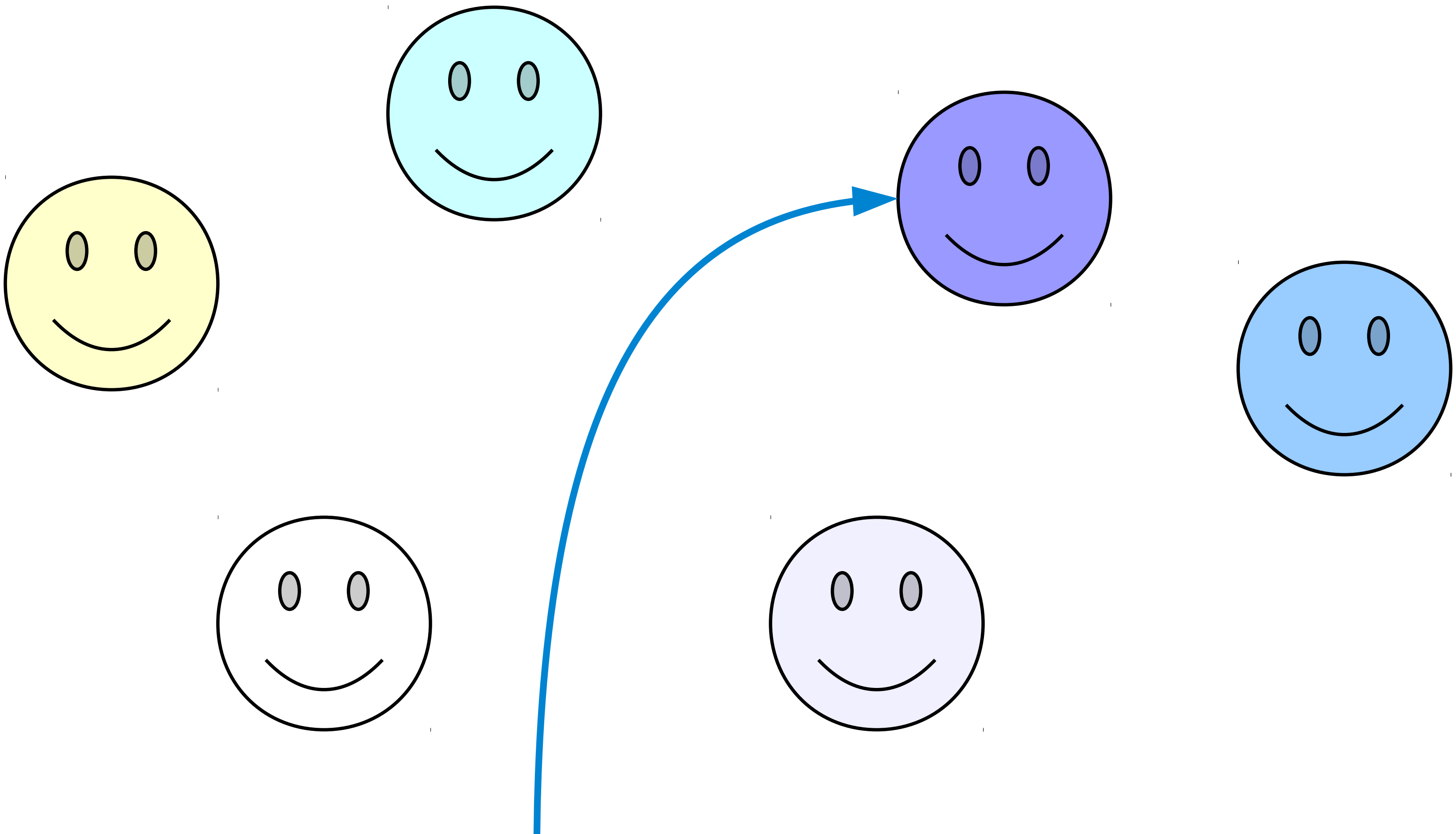

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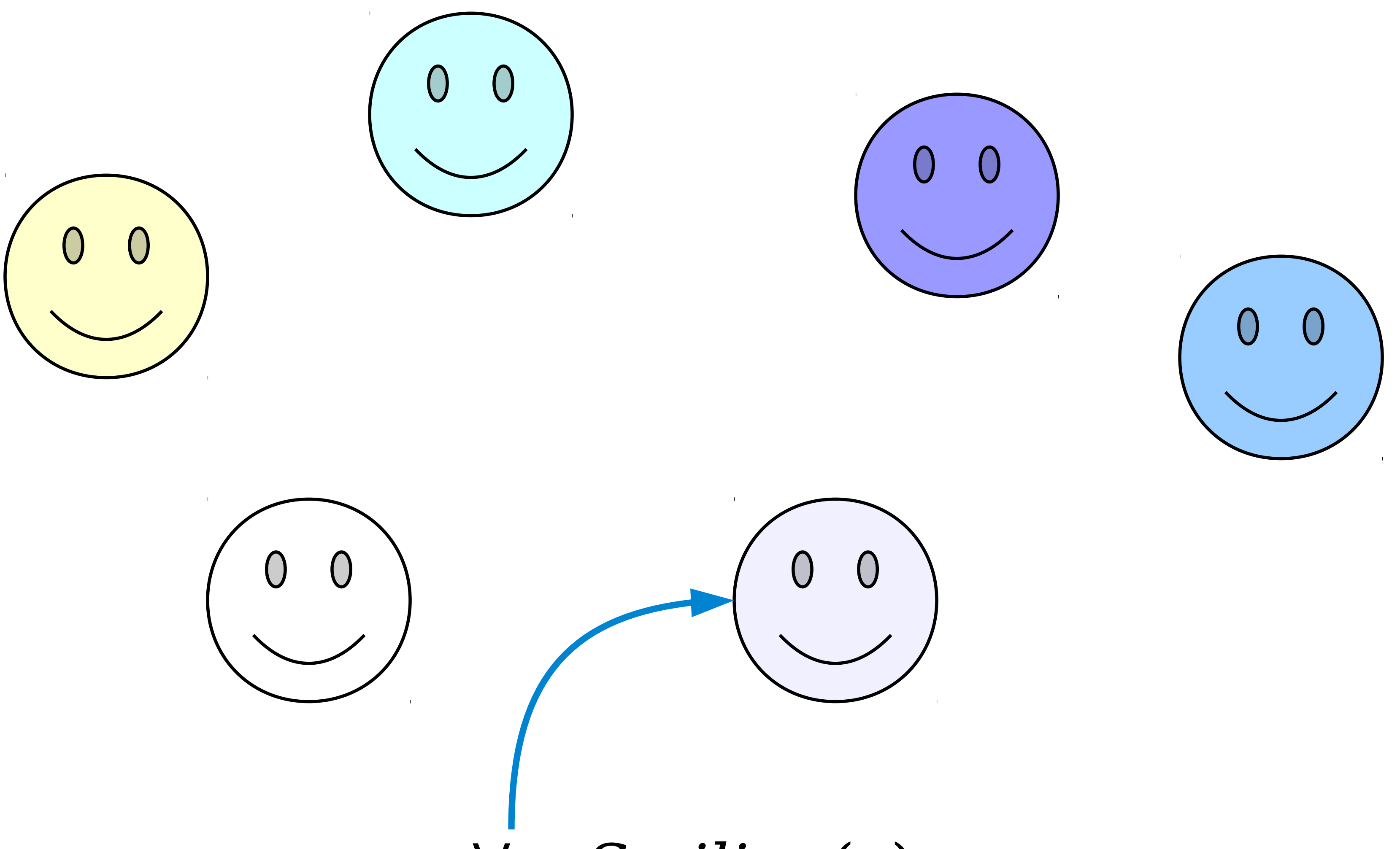

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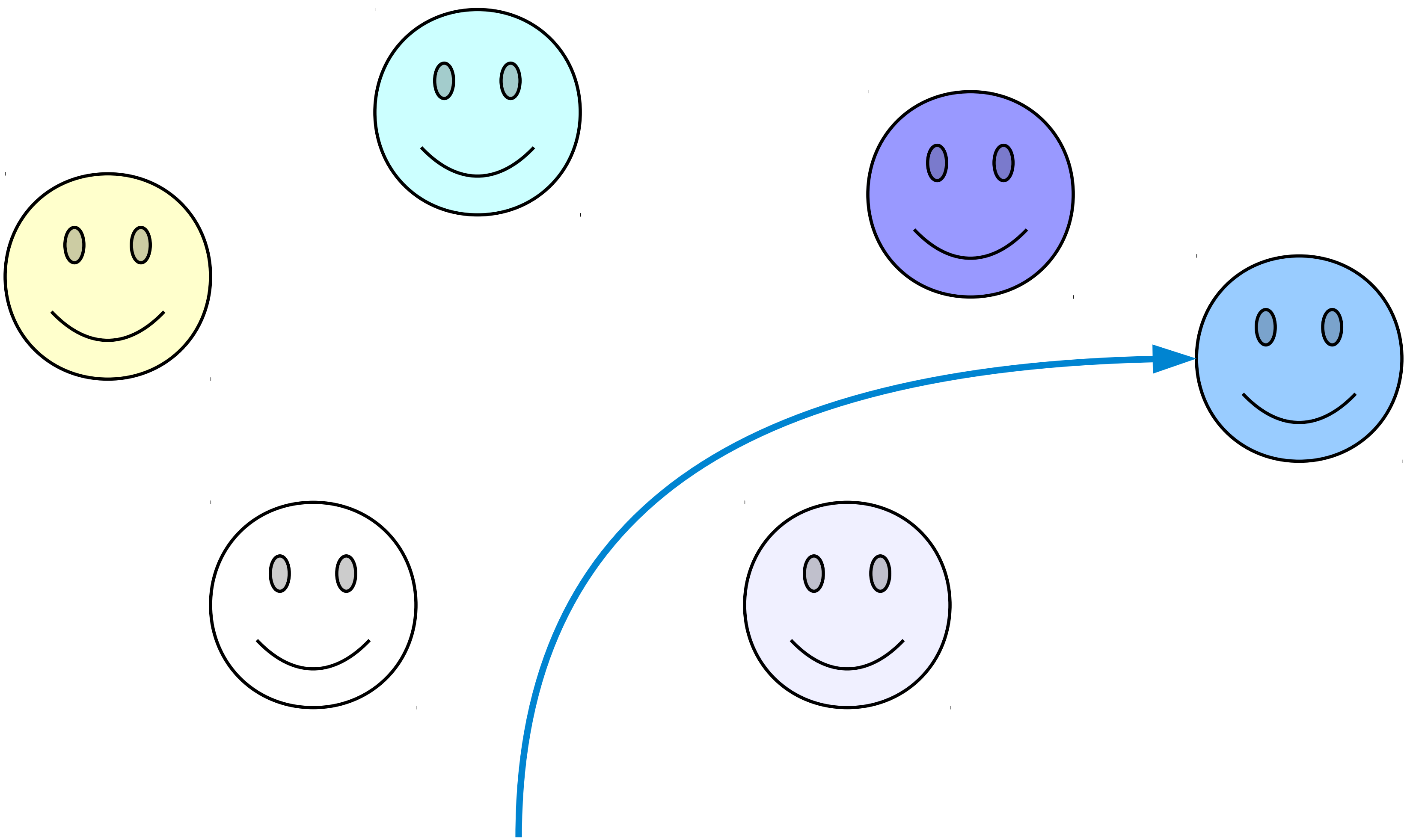

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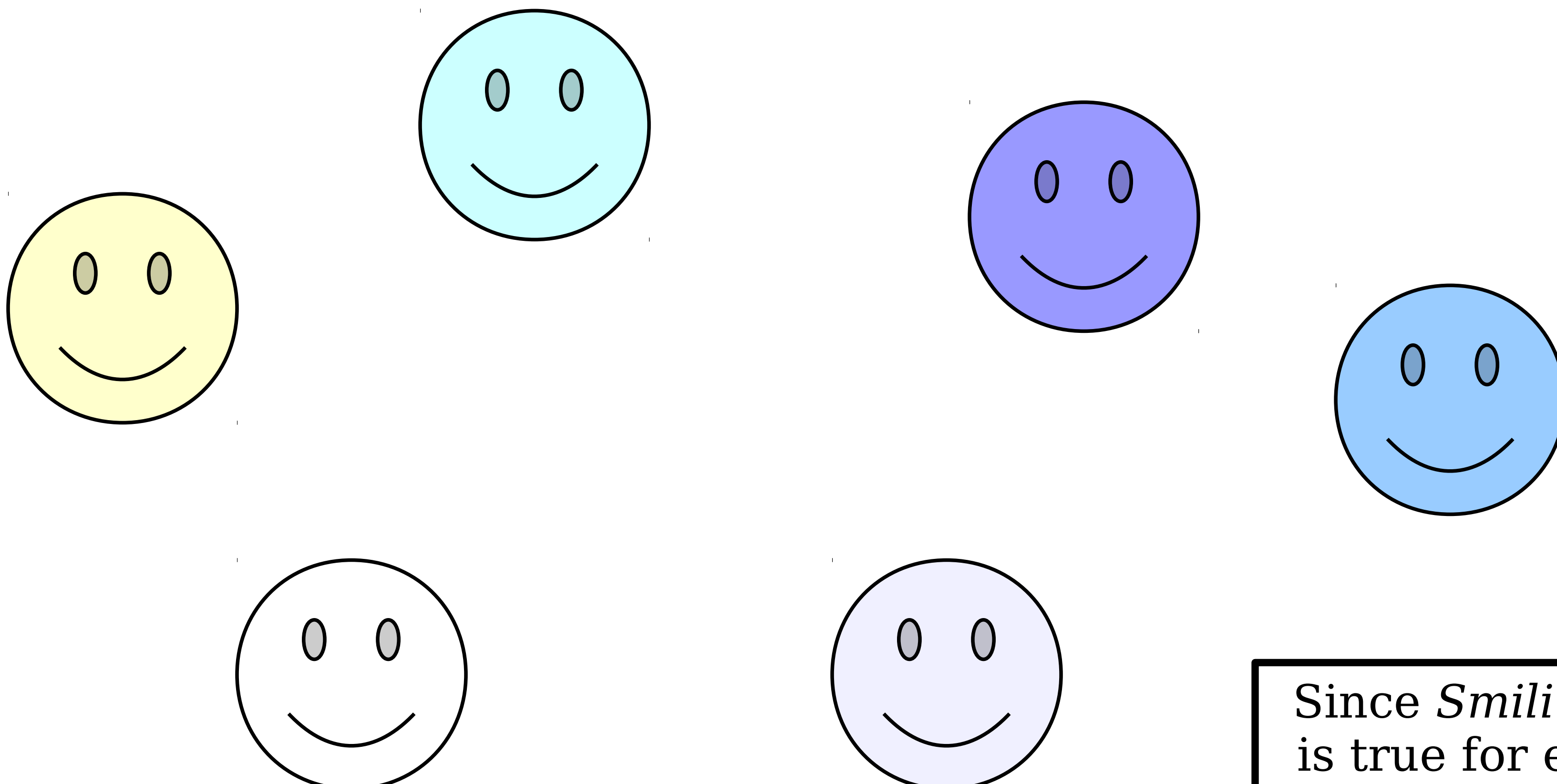
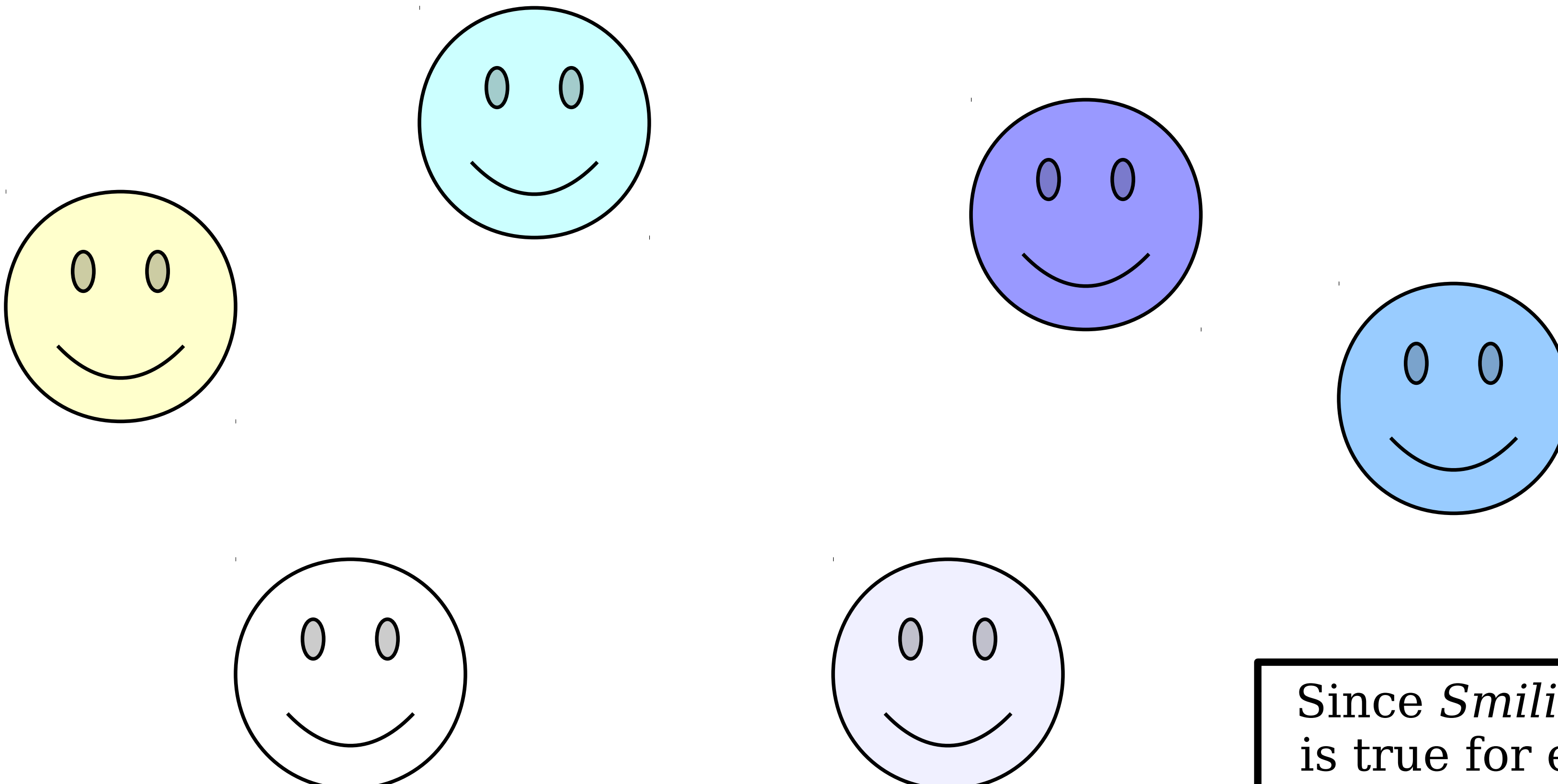


Since *Smiling*(x) is true for every choice of x, this statement evaluates to true.

$$\forall x.\ Smiling(x)$$

The Universal Quantifier



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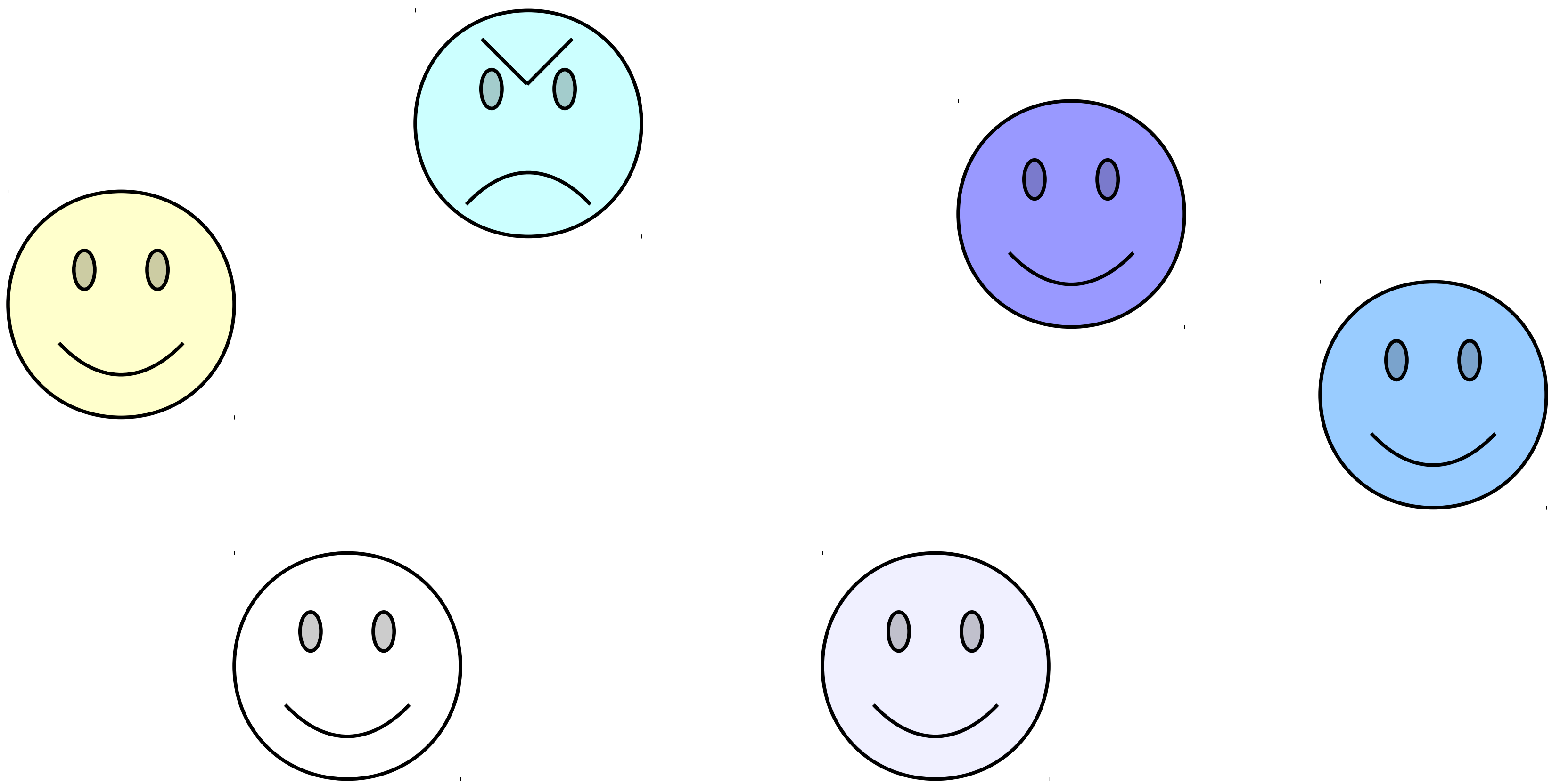

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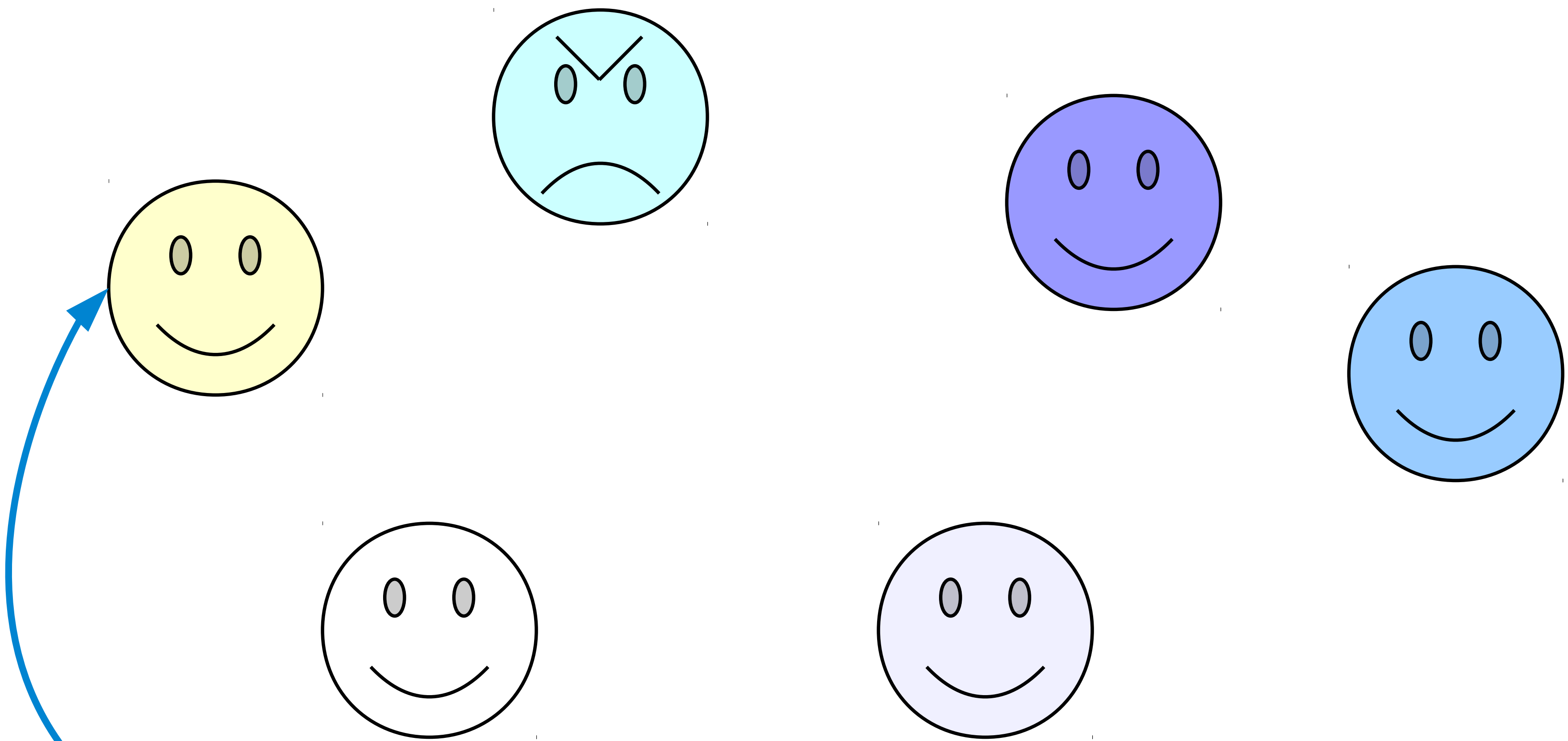

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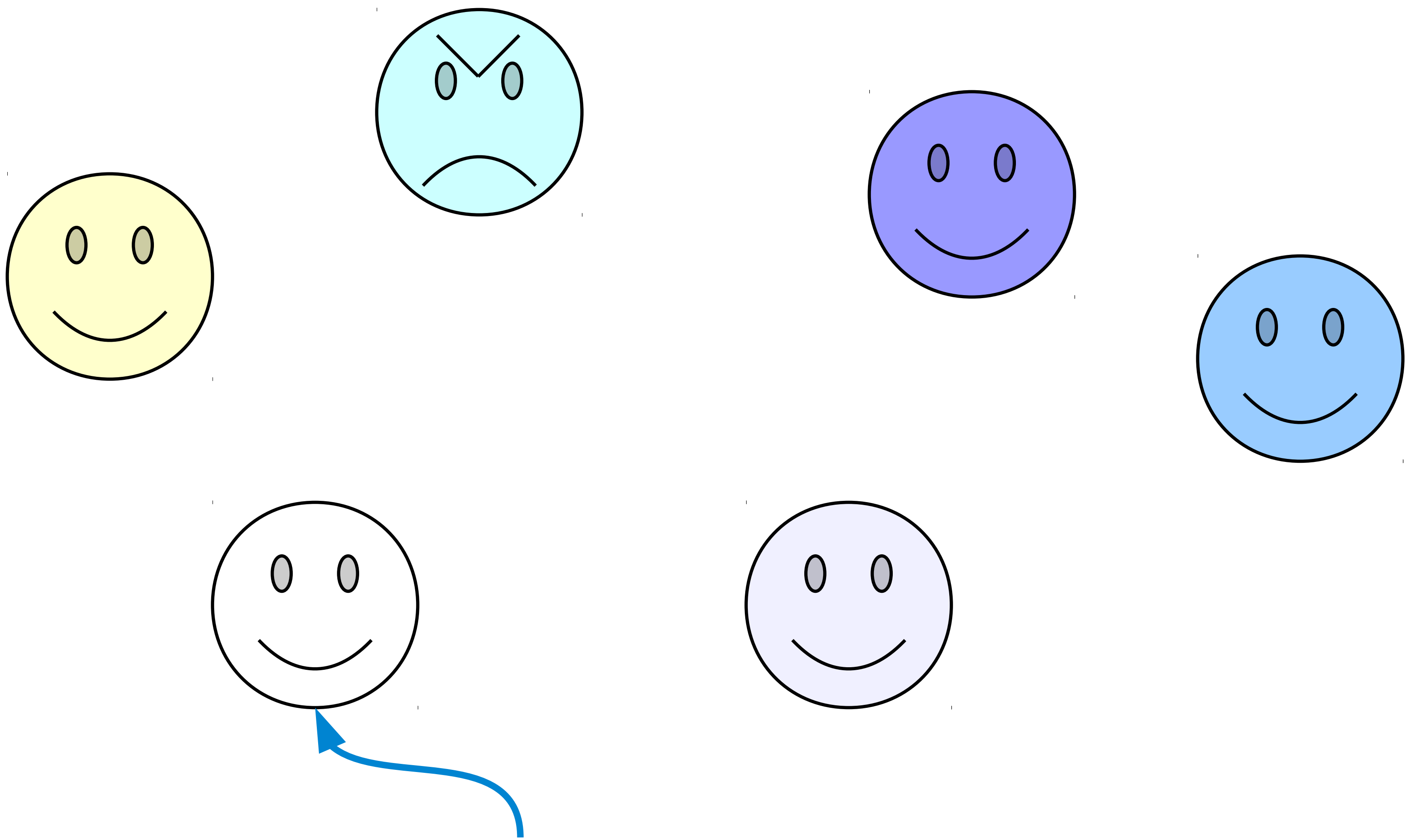

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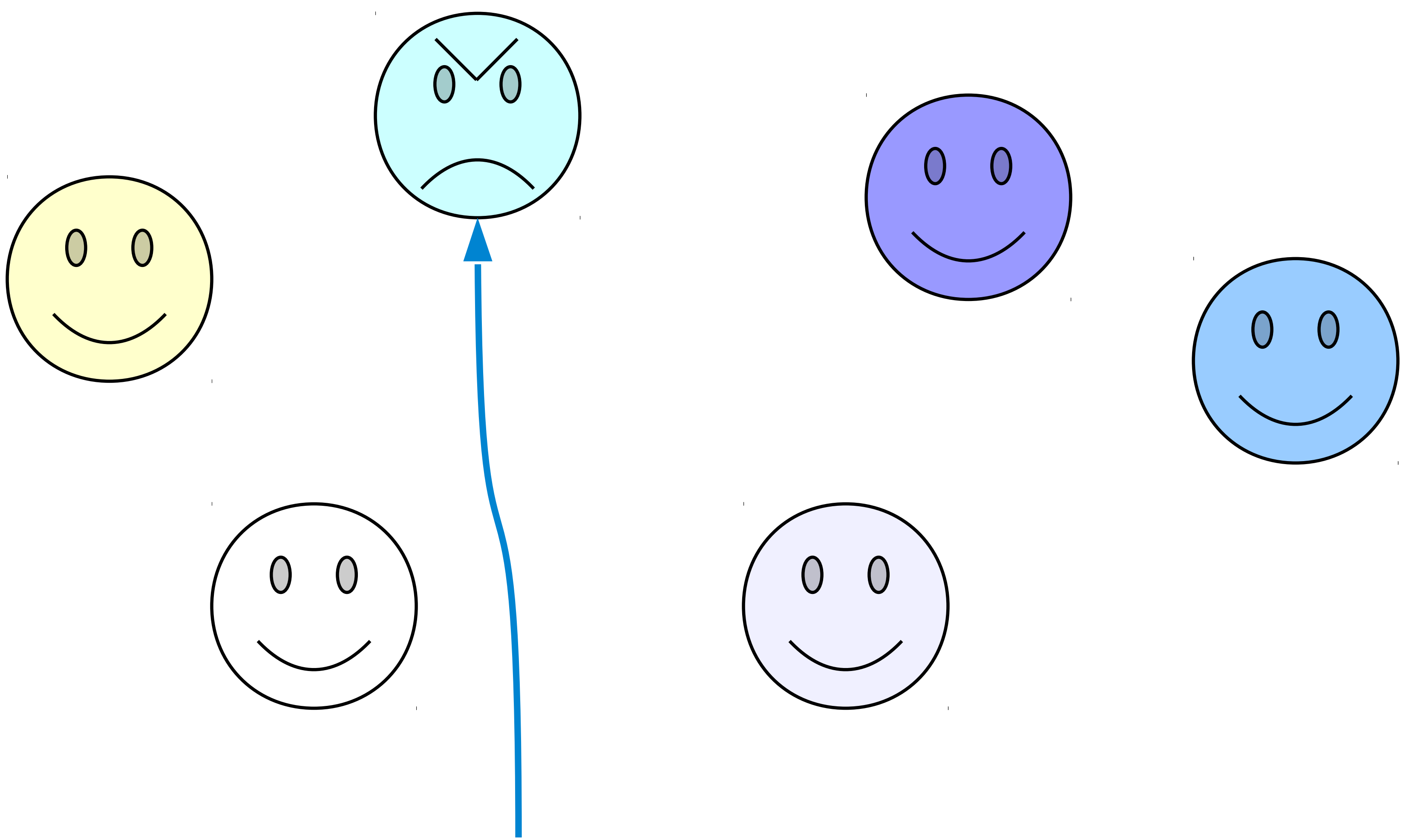

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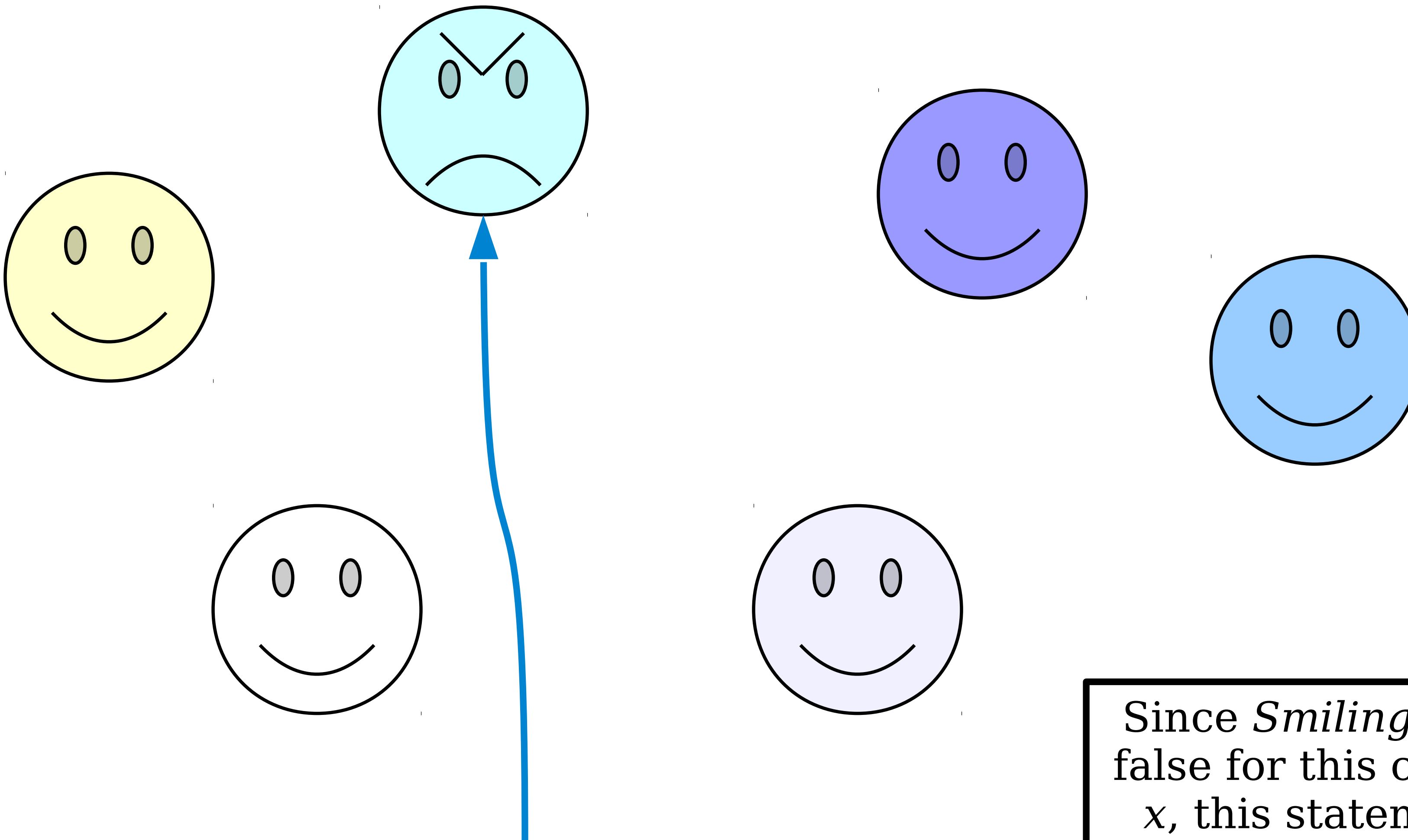
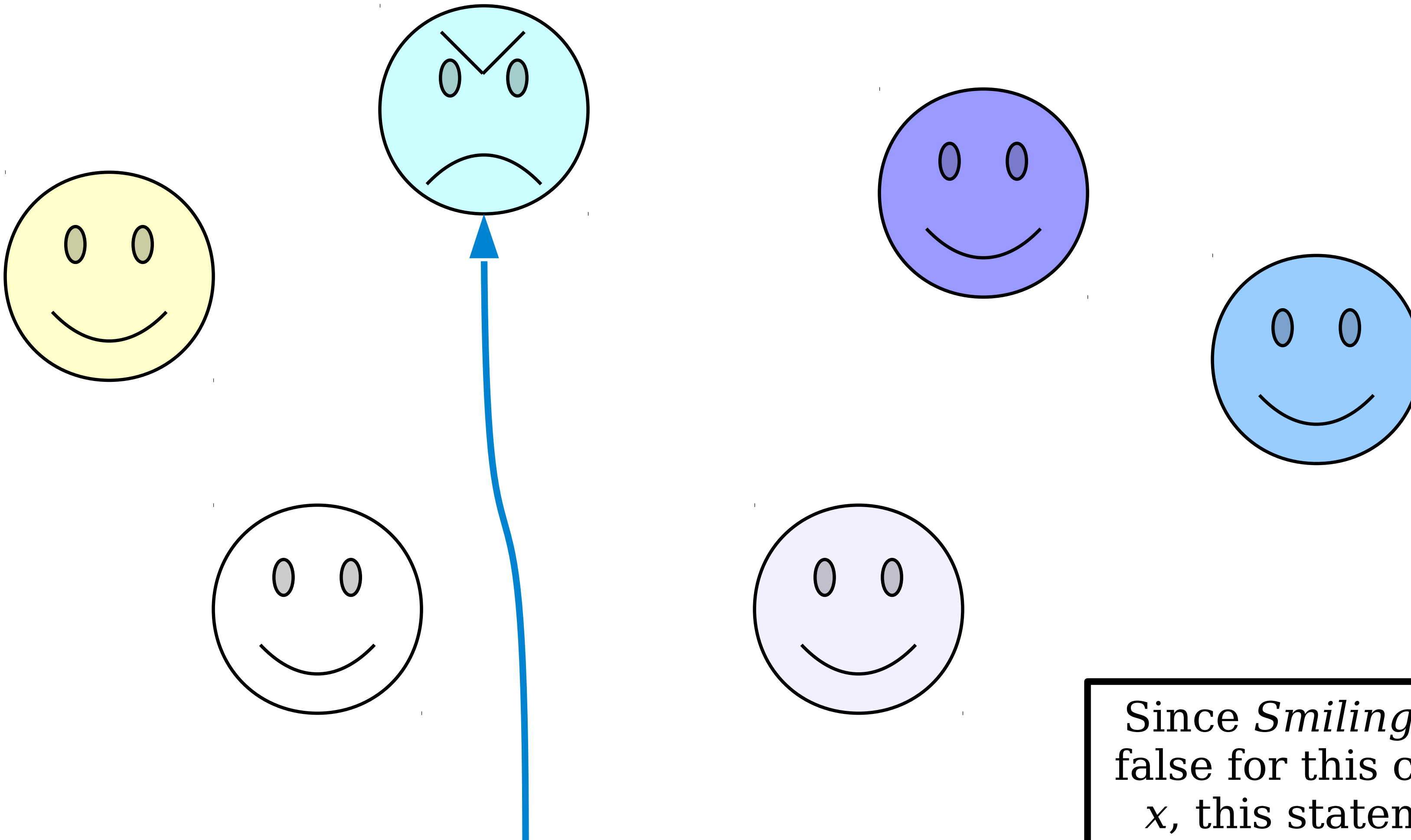


$\forall x.\ Smiling(x)$

Since *Smiling*(x) is false for this choice x, this statement evaluates to false.

The Universal Quantifier



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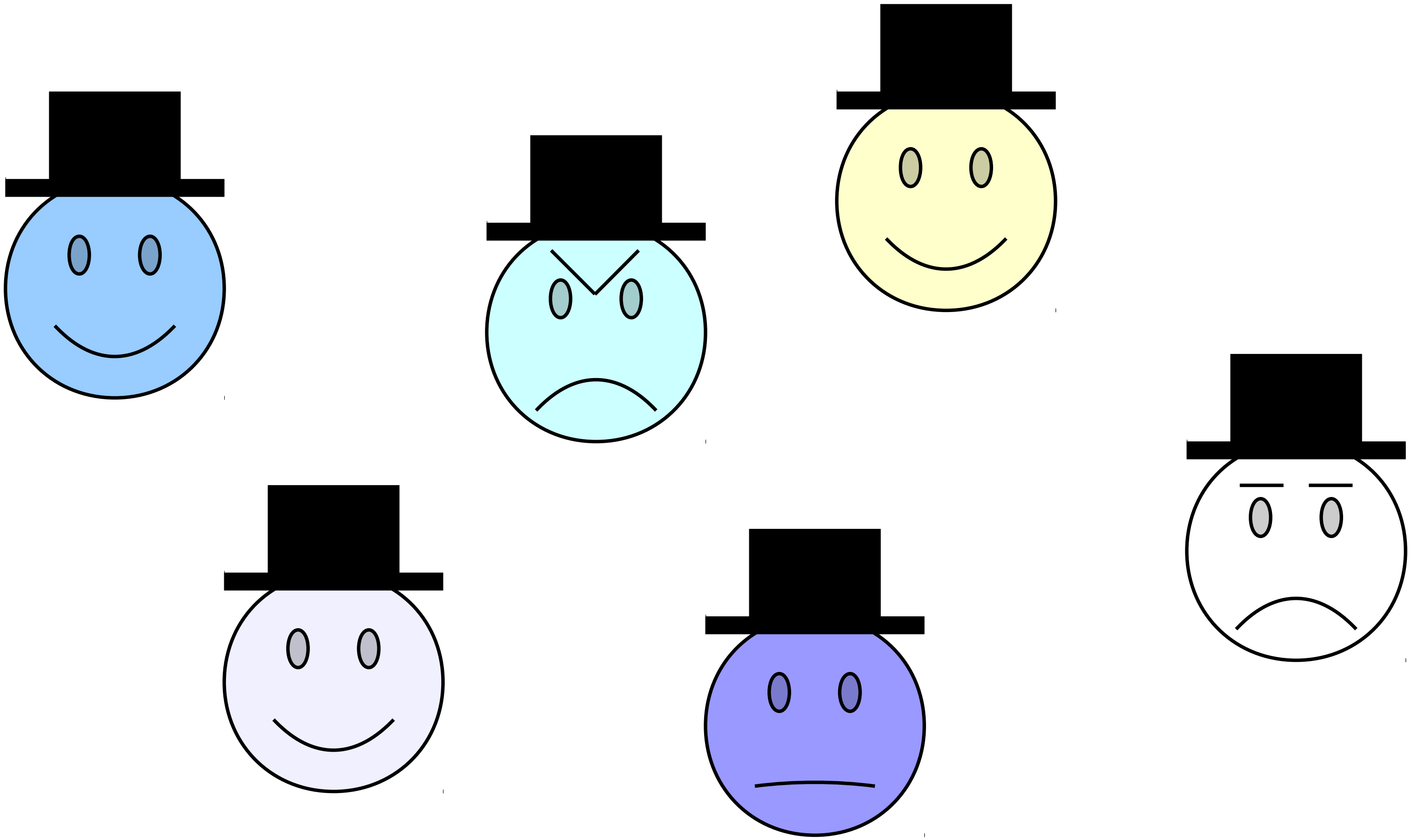
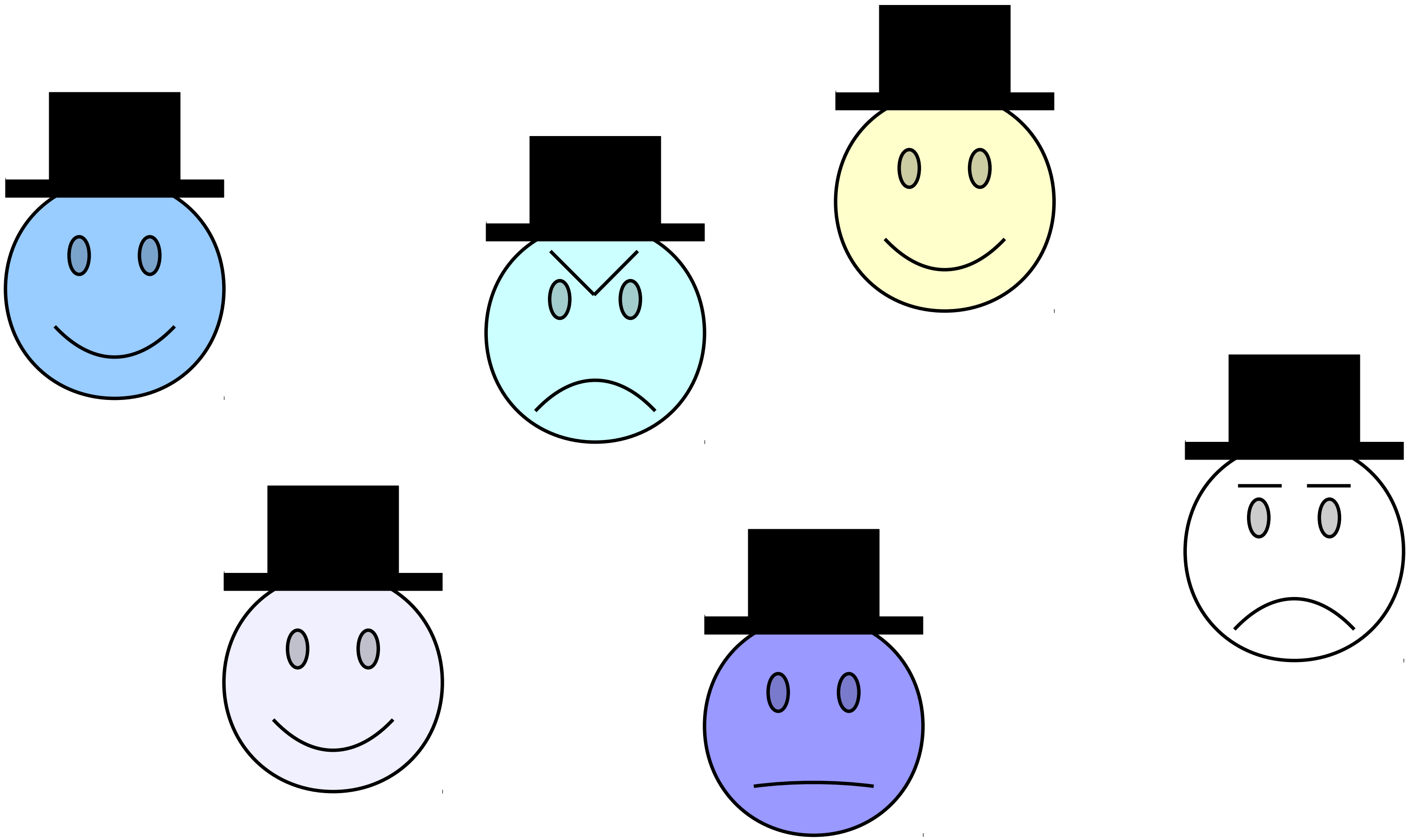


$$(\forall x.\ Smiling(x)) \rightarrow (\forall y.\ WearingHat(y))$$

The Universal Quantifier



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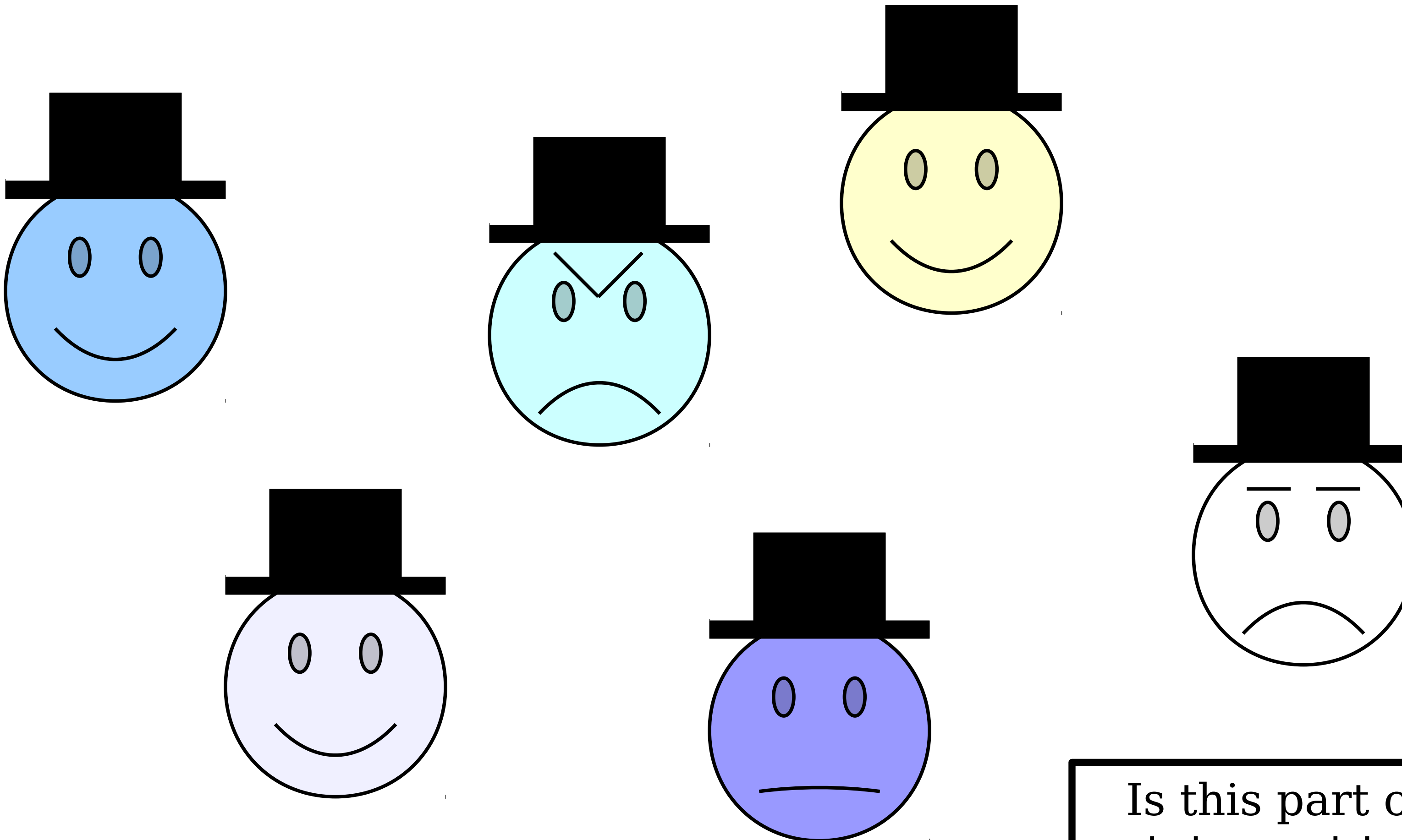
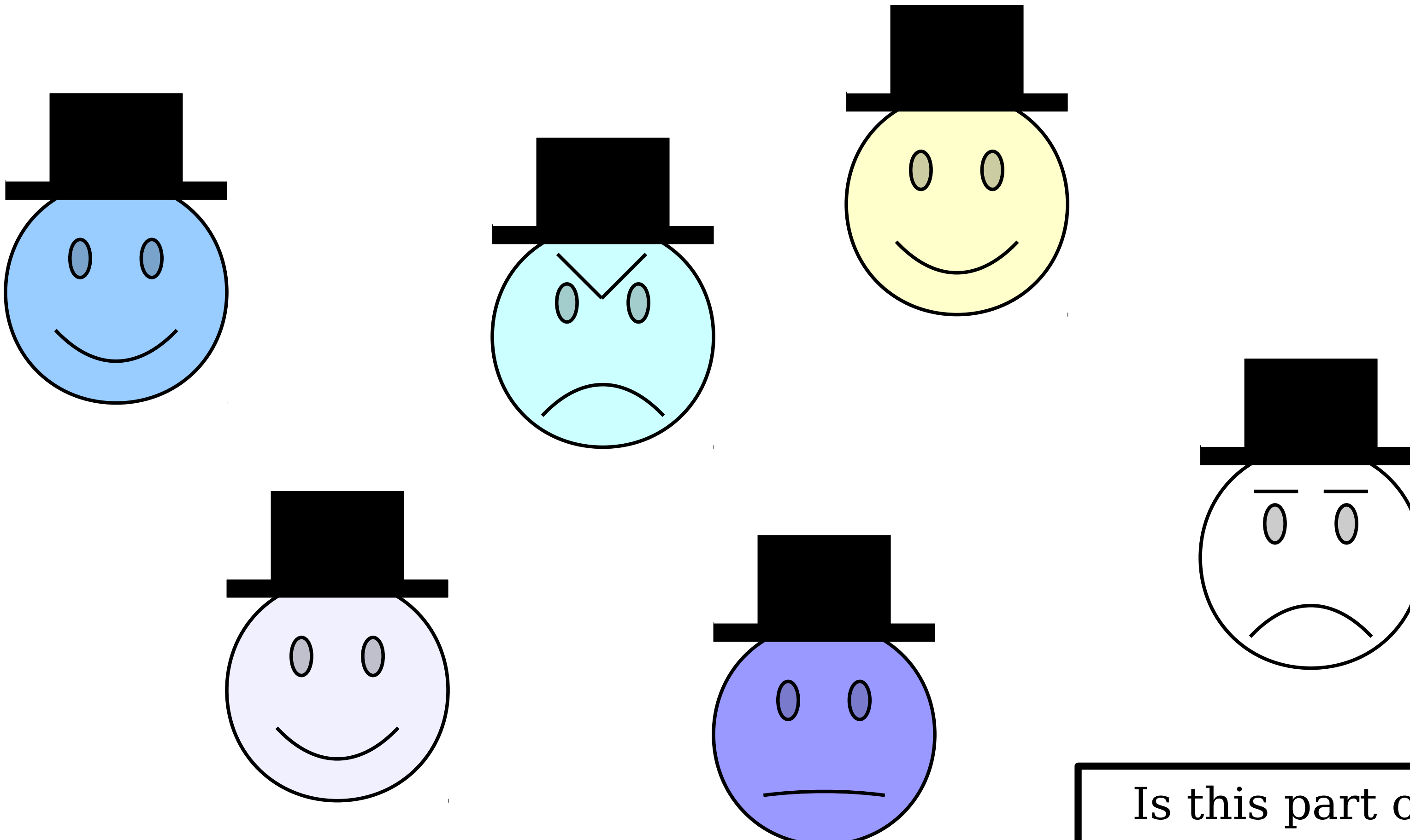


Is this part of the statement true or false?

$(\forall x.\ Smiling(x)) \rightarrow (\forall y.\ WearingHat(y))$

The Universal Quantifier



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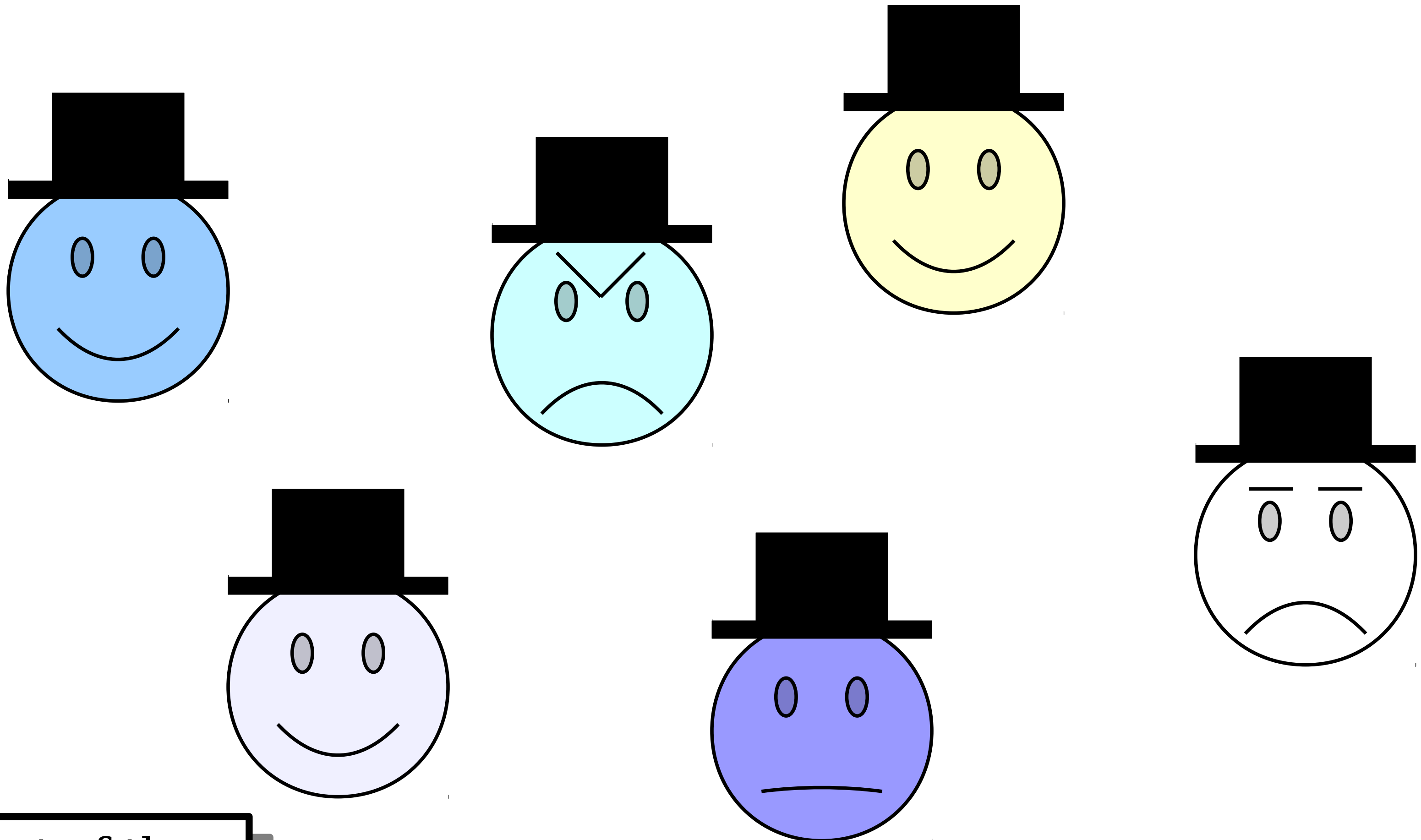

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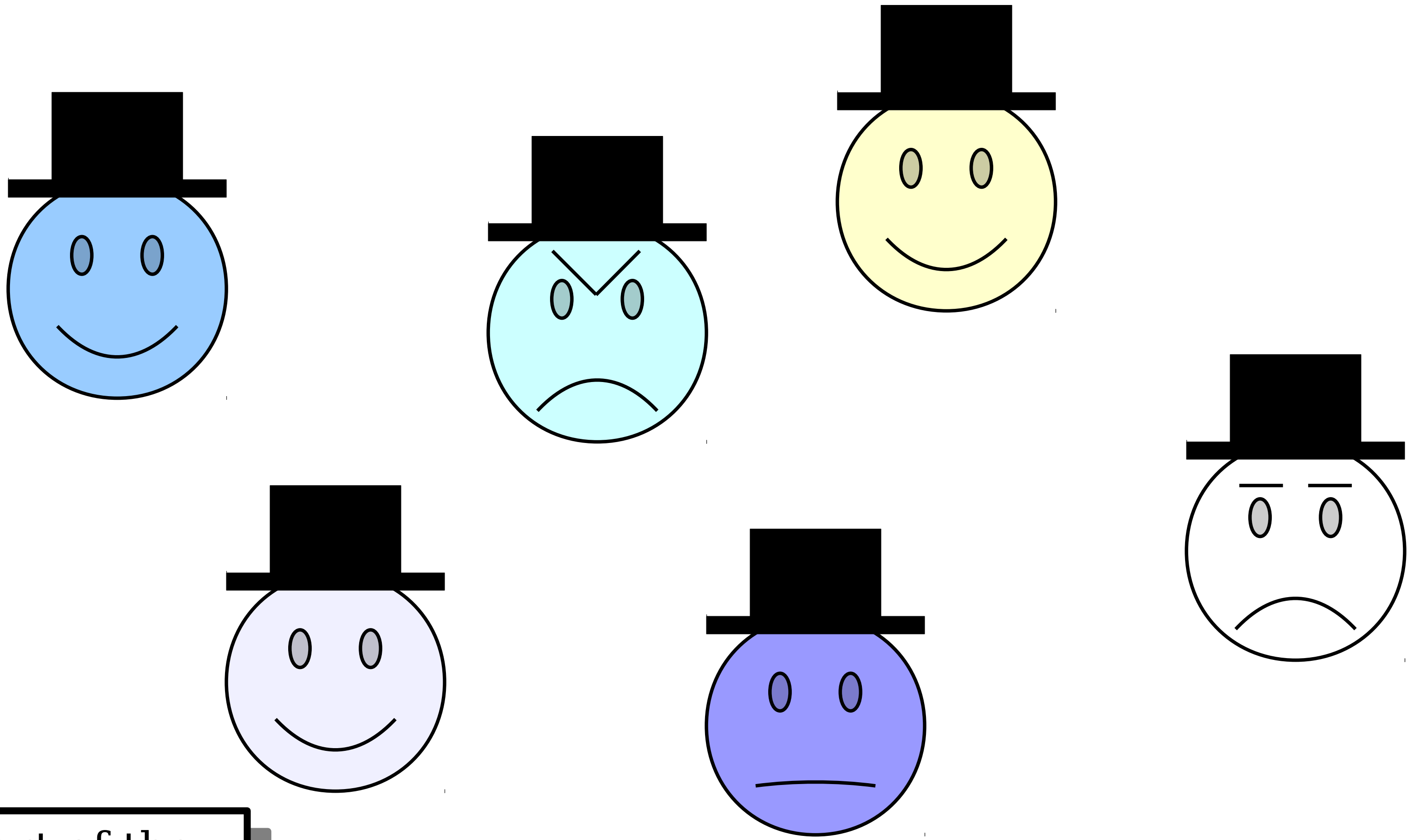

Is this part of the statement true or false?

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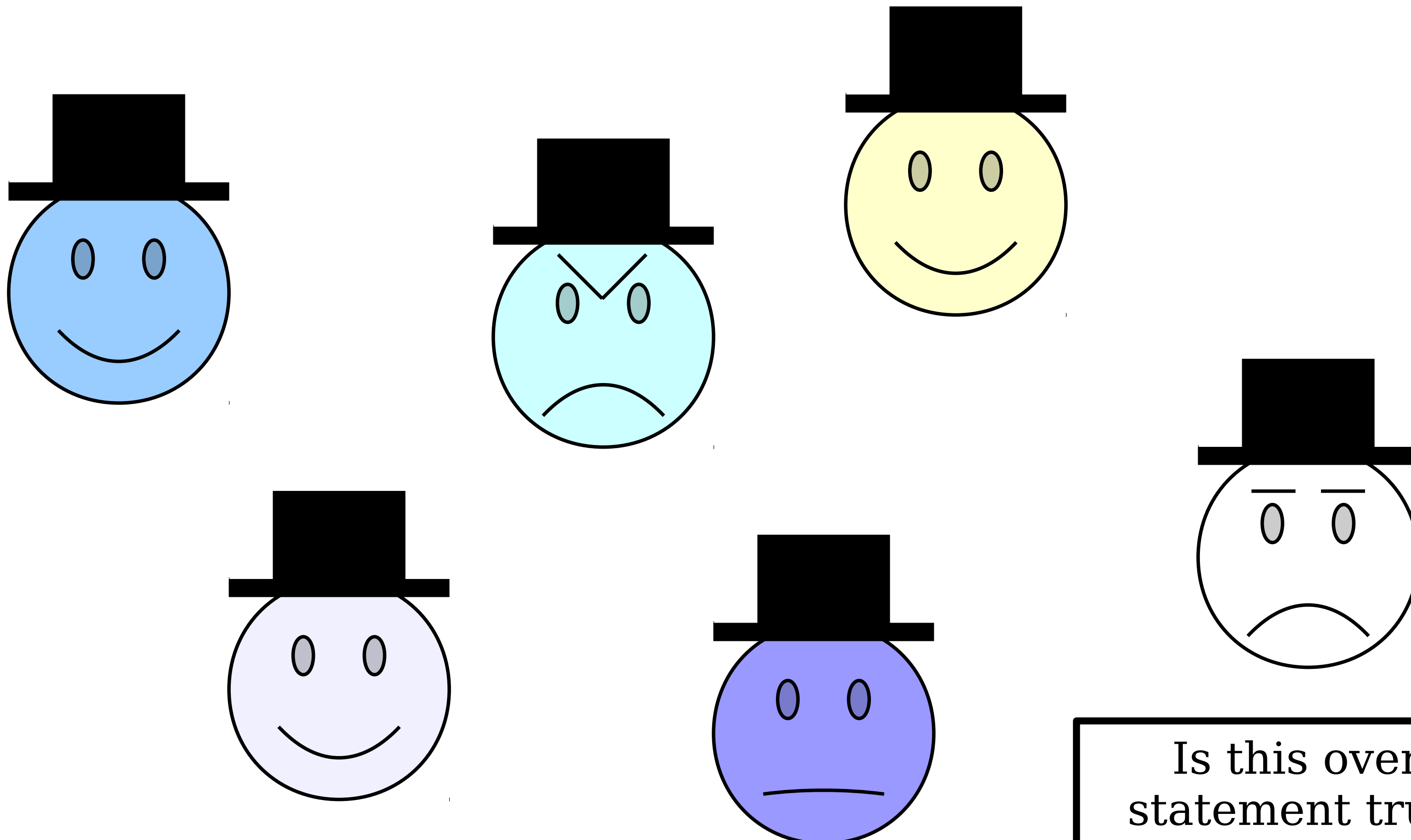
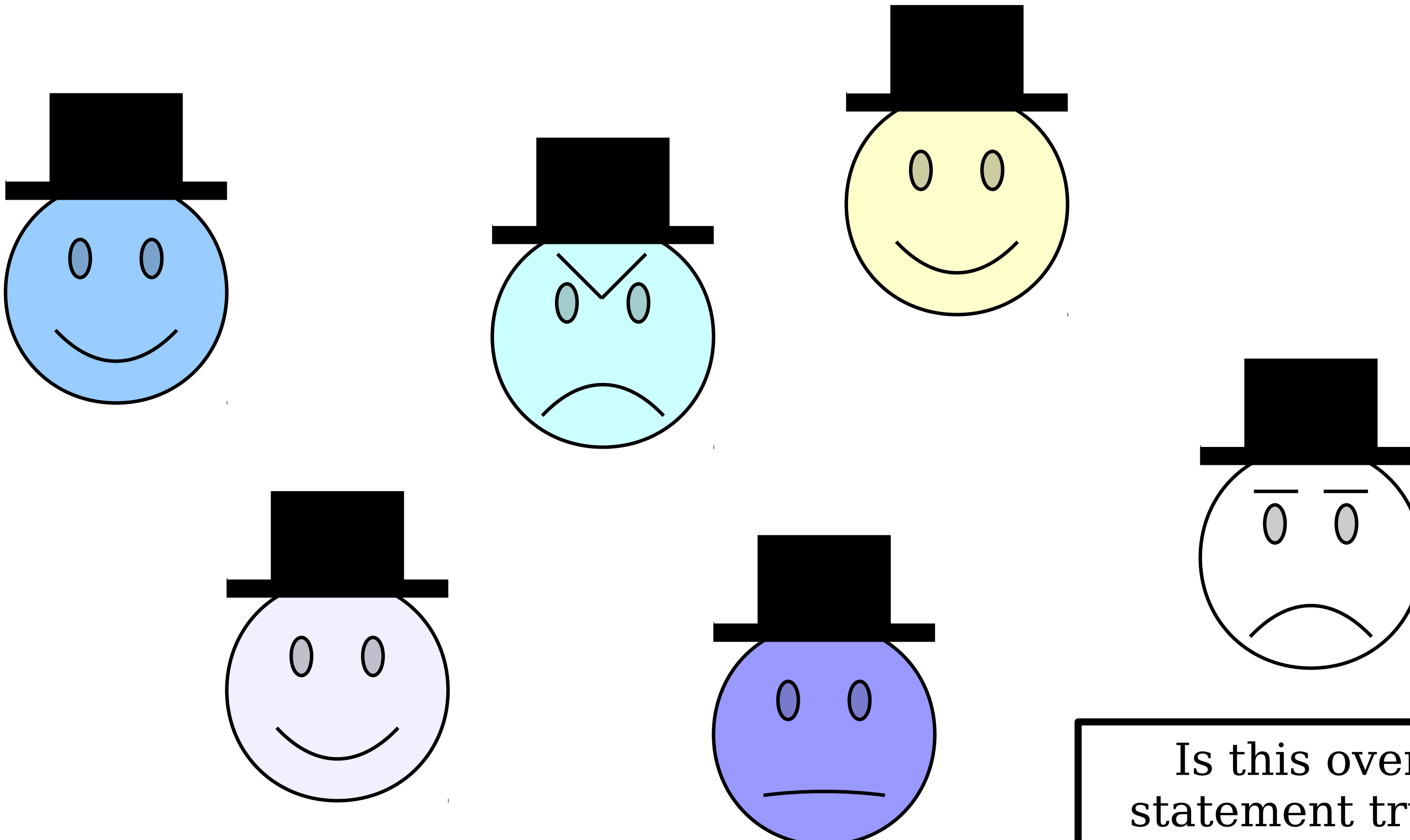


Is this overall statement true or false in this scenario?

~~$(\forall x.\ Smiling(x))$~~ $\to (\forall y.\ WearingHat(y))$

The Universal Quantifier



Is this overall statement true or false in this scenario?

$$(\forall x.\ Smiling(x)) \to (\forall y.\ WearingHat(y))$$

Fun with Edge Cases

$$\forall x.\ Smiling(x)$$

Fun with Edge Cases

Universally-quantified statements are ***vacuously true*** in empty worlds.

$$\forall x.\ Smiling(x)$$

Translating into First-Order Logic

Translating Into Logic

- First-order logic is an excellent tool for manipulating definitions and theorems to learn more about them.
- Need to take a negation? Translate your statement into FOL, negate it, then translate it back.
- Want to prove something by contrapositive? Translate your implication into FOL, take the contrapositive, then translate it back.

Translating Into Logic

- ***Translating statements into first-order logic is a lot more difficult than it looks.***
- There are a lot of nuances that come up when translating into first-order logic.
- We'll cover examples of both good and bad translations into logic so that you can learn what to watch for.
- We'll also show lots of examples of translations so that you can see the process that goes into it.

Using the predicates

- $Puppy(p)$, which states that p is a puppy, and
- $Cute(x)$, which states that x is cute,

write a sentence in first-order logic that means “all puppies are cute.”

An Incorrect Translation

All puppies are cute!

$$\forall x.\ (Puppy(x) \land Cute(x))$$

An Incorrect Translation

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$$\forall x.\ (Puppy(x) \land Cute(x))$$

This should work for <u>any</u> choice of x, including things that aren't puppies.

An Incorrect Translation

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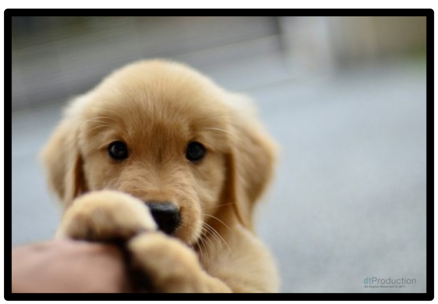
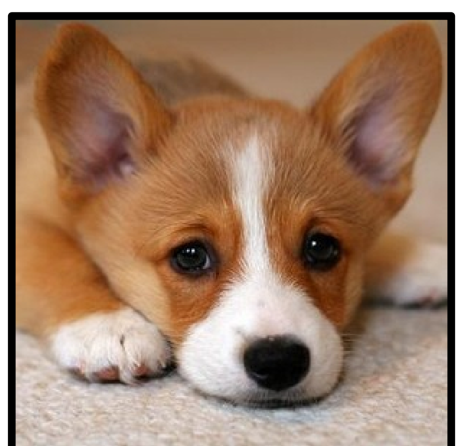


$$\forall x. \; (\mathit{Puppy}(x) \land \mathit{Cute}(x))$$



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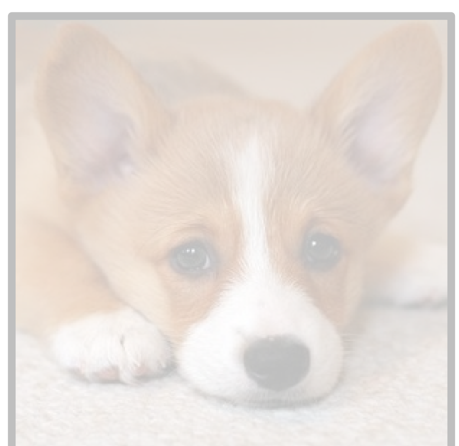

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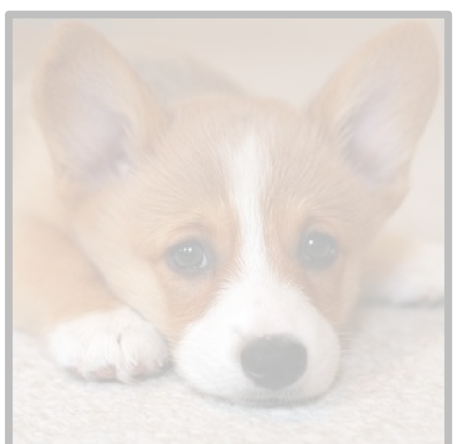
All puppies are cute!

$\forall x. ($~~$Puppy(x)$~~$\land Cute(x))$



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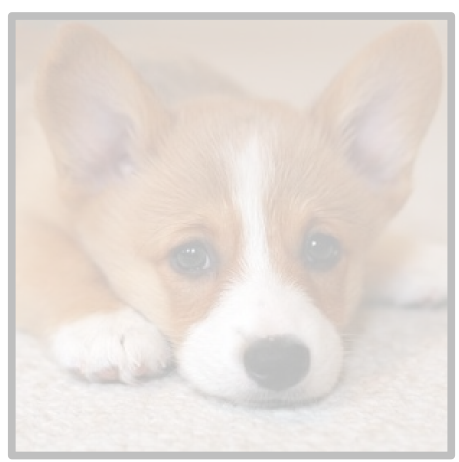
All puppies are cute!

$\forall x.$ (~~$Puppy(x) \land Cute(x)$~~)



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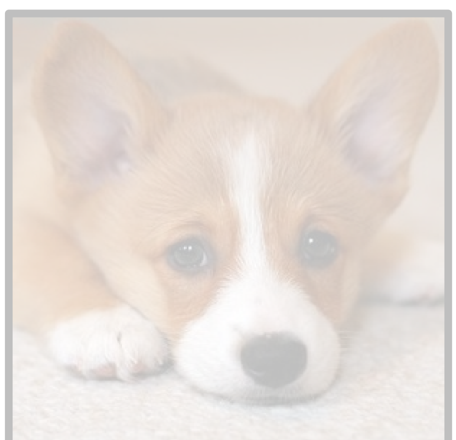

A statement of the form

$\forall x.$ ***something***

is true only when *something* is true for *every* choice of x.

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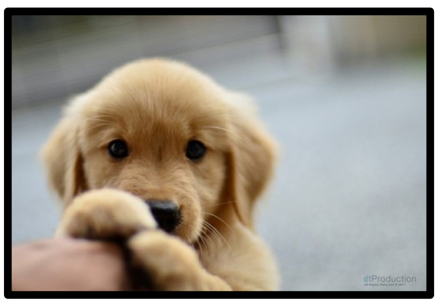

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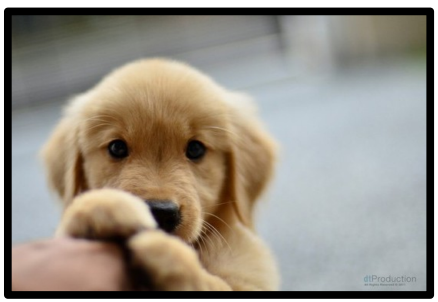

~~$\forall x. (Puppy(x) \land Cute(x))$~~

This first-order statement is false even though the English statement is true. Therefore, it can't be a correct translation.

An Incorrect Translation

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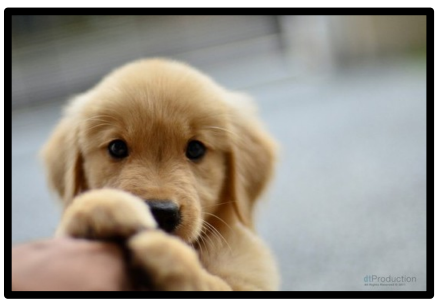

$\forall x.\ (Puppy(x) \land Cute(x))$

The issue here is that this statement asserts that everything is a puppy. That's too strong of a claim to make.

A Better Translation

All puppies are cute!

$$\forall x.\ (Puppy(x) \rightarrow Cute(x))$$

A Better Translation

All puppies are cute!

$$\forall x.\ (Puppy(x) \to Cute(x))$$

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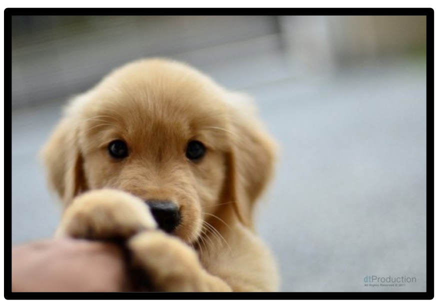

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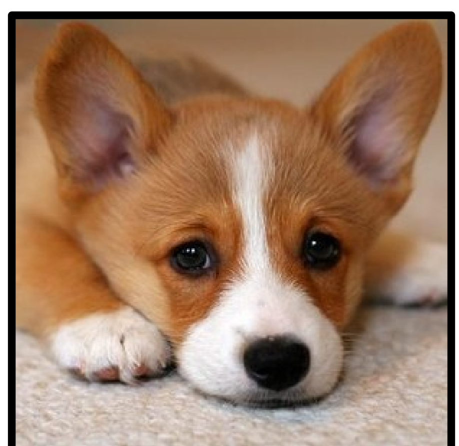

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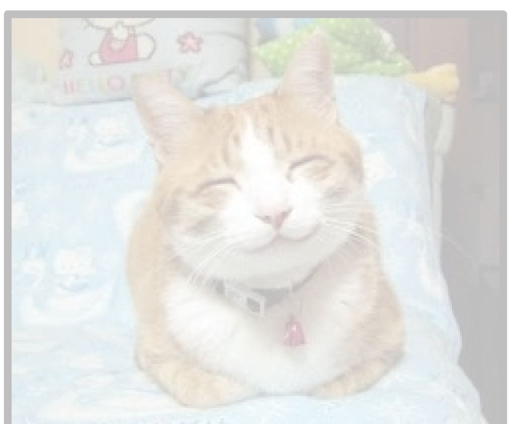

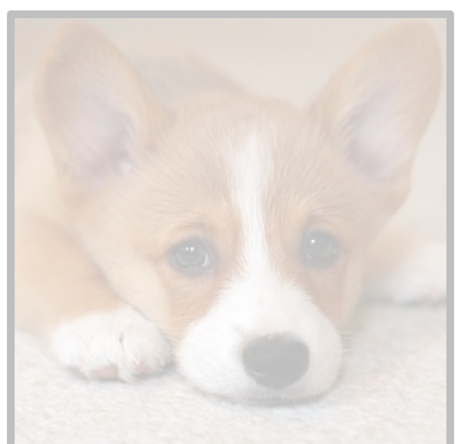

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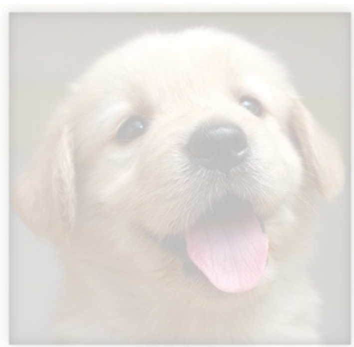

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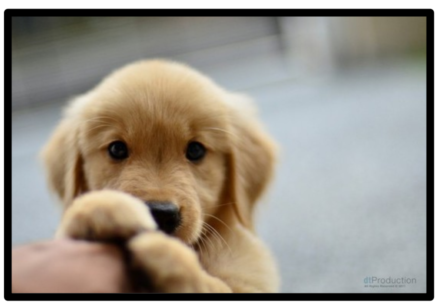

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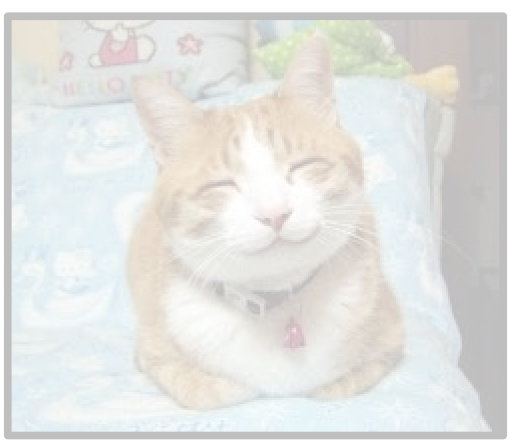

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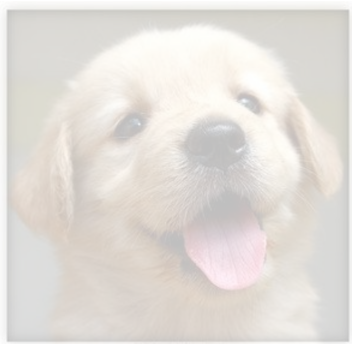

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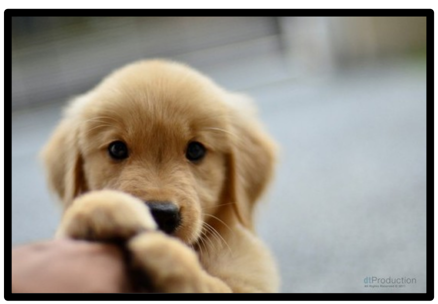

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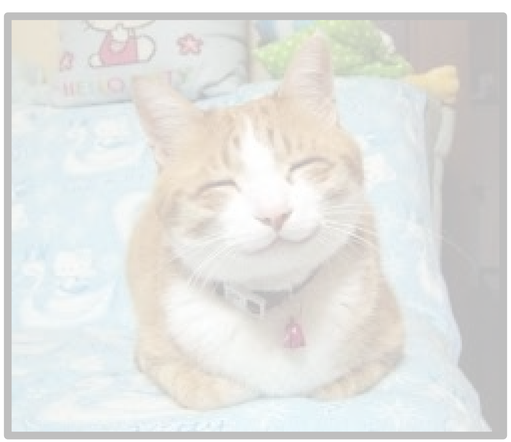

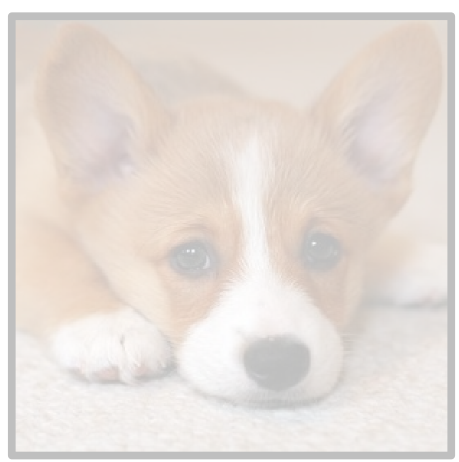

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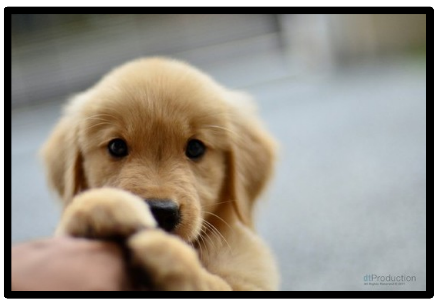

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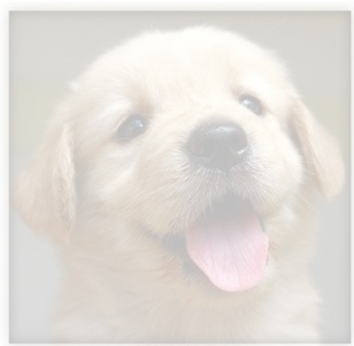

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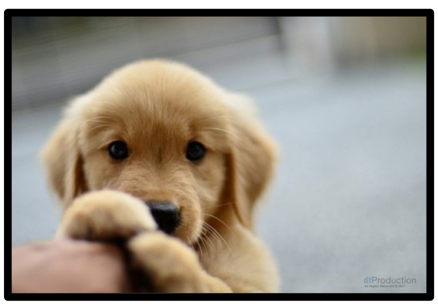

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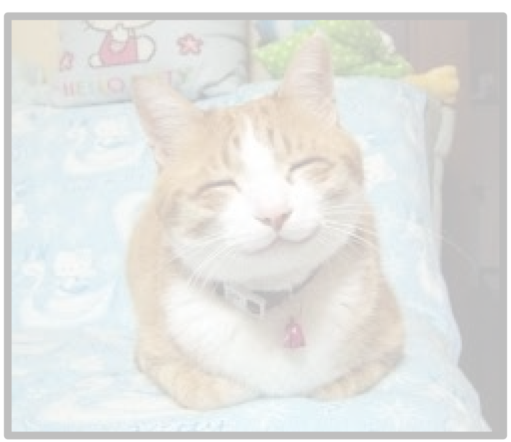

This should work for *any* choice of x, including things that aren't puppies.

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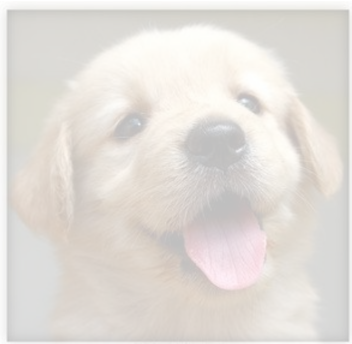

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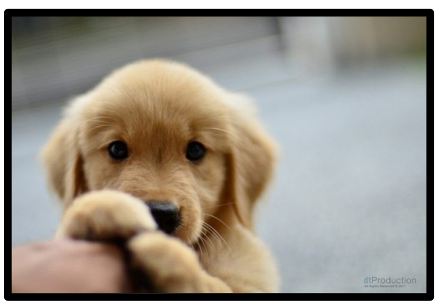

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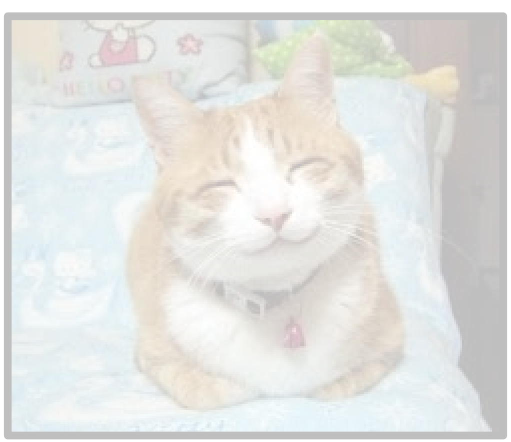

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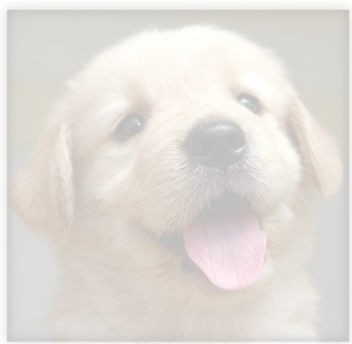

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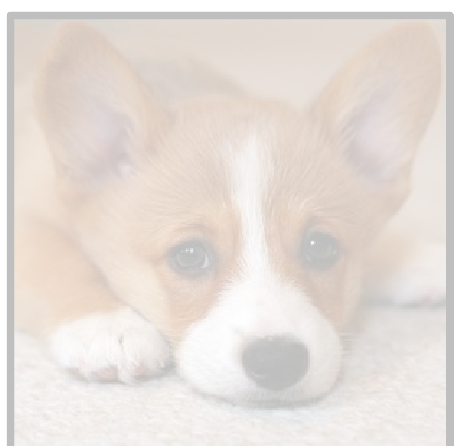

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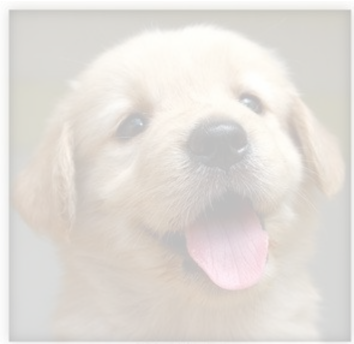

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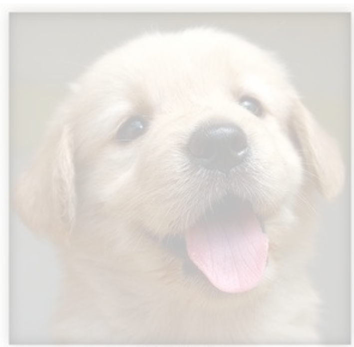

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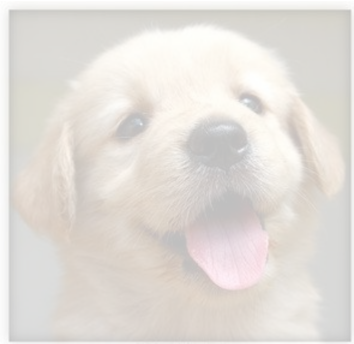

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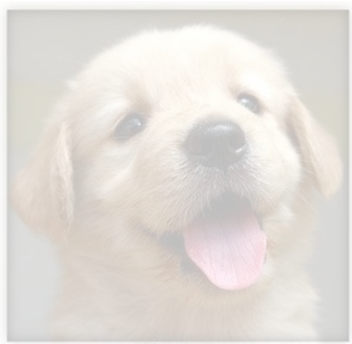

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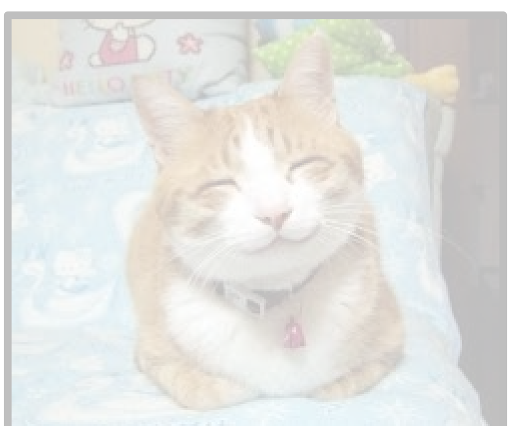

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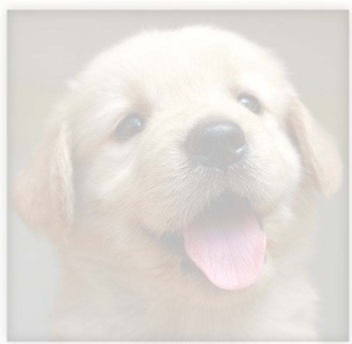

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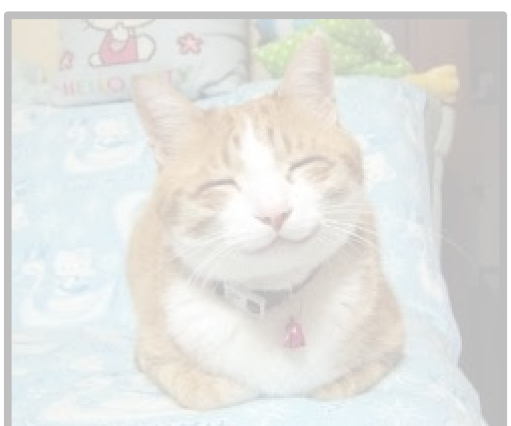

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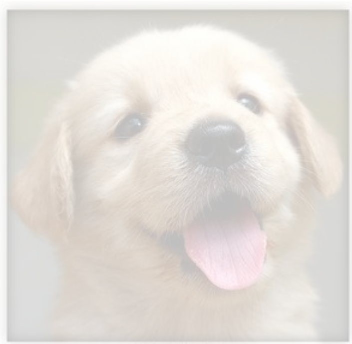

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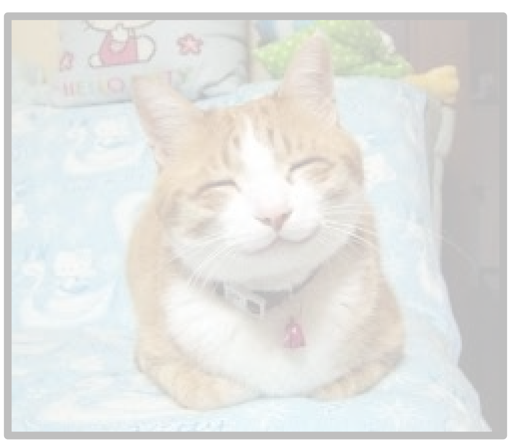

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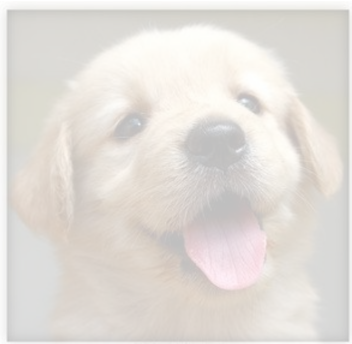

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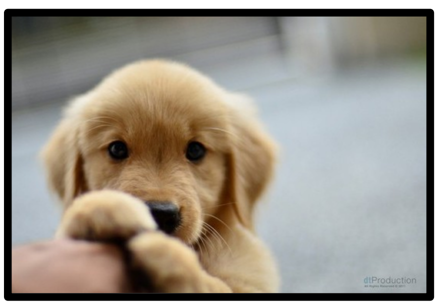

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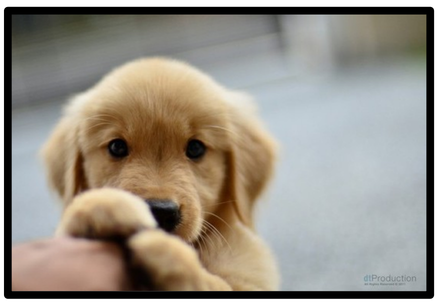

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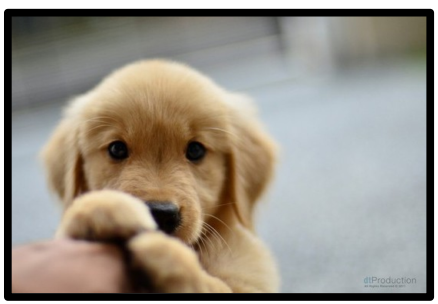

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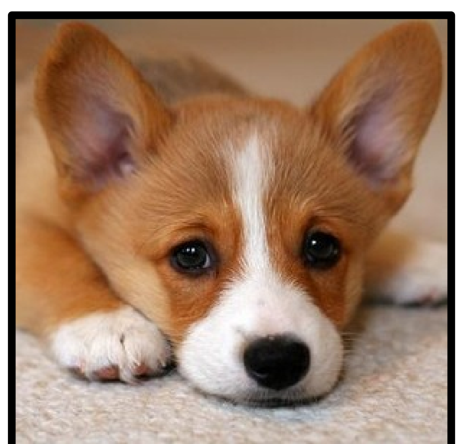

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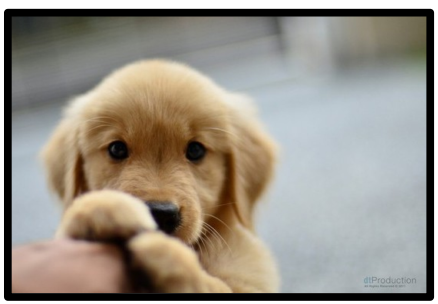

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A statement of the form

$\forall$x. ***something***

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"All *P*'s are *Q*'s"

translates as

$$\forall x.\ (P(x) \to Q(x))$$

Useful Intuition:

Universally-quantified statements are true unless there's a counterexample.

$$\forall x. (P(x) \to Q(x))$$

If x is a counterexample, it *must* have property P but not have property Q.

Time-Out for Announcements!

Checkpoints Graded

- The Problem Set One checkpoint problem has been graded. Feedback is now available in GradeScope.
- ***You need to look over our feedback as soon as possible.***
  - The purpose of the checkpoint is to help you see where to focus and how to improve.
  - If you don't review the feedback you received, you risk making the same mistakes in the future.

Back to CS103!

Using the predicates

- *Blobfish*(b), which states that b is a blobfish, and
- *Cute*(x), which states that x is cute,

write a sentence in first-order logic that means “some blobfish is cute.”

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- $Blobfish(b)$, which states that b is a blobfish, and
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An Incorrect Translation

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$$\exists x.\ (Blobfish(x) \rightarrow Cute(x))$$

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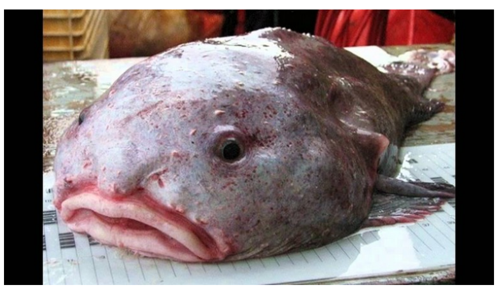

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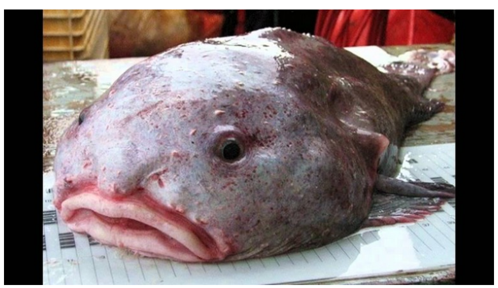

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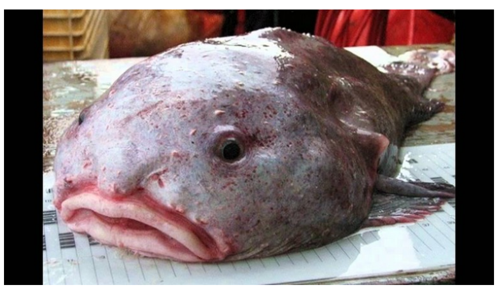

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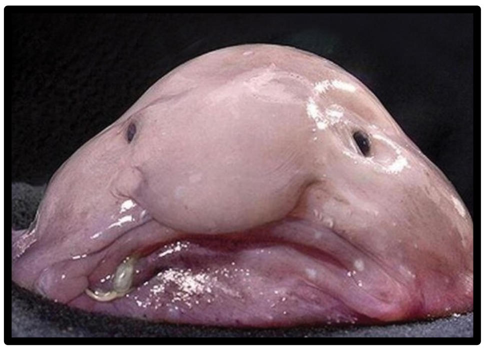
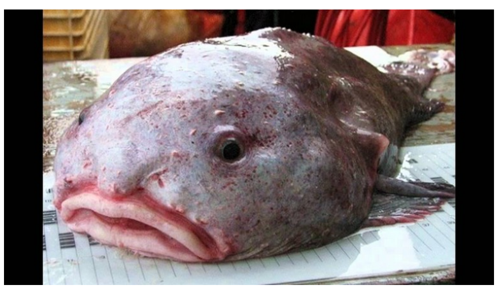
$\exists x. (Blobfish(x) \rightarrow Cute(x))$

This first-order statement is true even though the English statement is false. Therefore, it can't be a correct translation.

An Incorrect Translation





Some blobfish is cute.

$$\exists x.\ (\mathit{Blobfish}(x) \to \mathit{Cute}(x))$$

The issue here is that implications are true whenever the antecedent is false. This statement "accidentally" is true because of what happens when x isn't a blobfish.

A Correct Translation

Some blobfish is cute.

$$\exists x.\ (\mathit{Blobfish}(x) \land \mathit{Cute}(x))$$

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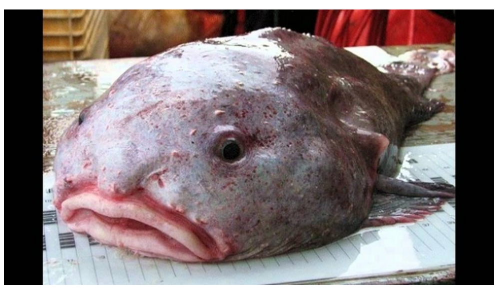

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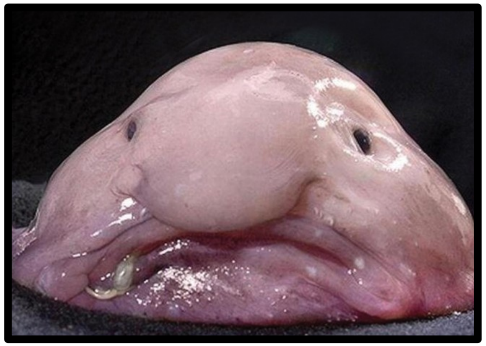

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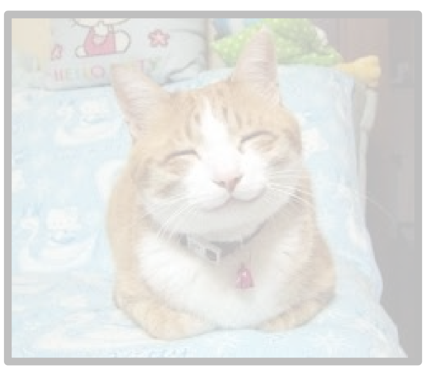

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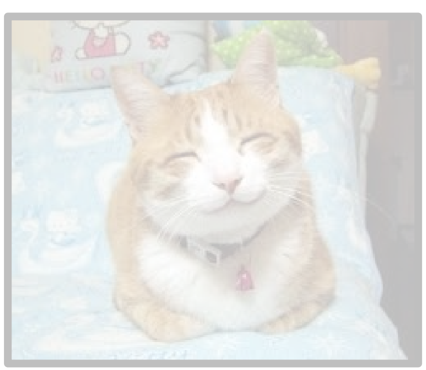

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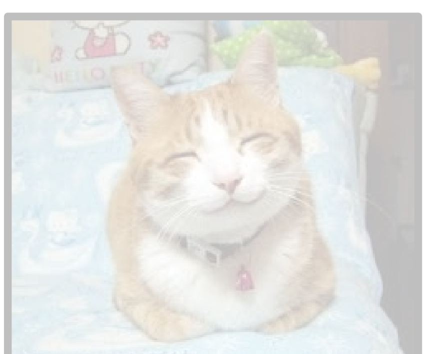

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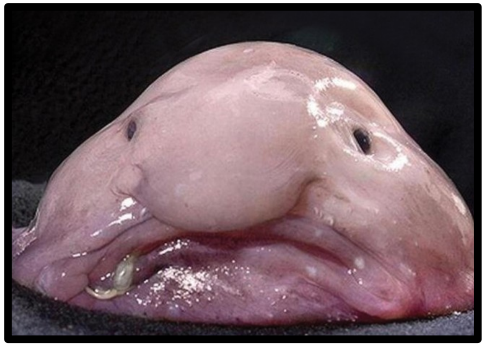

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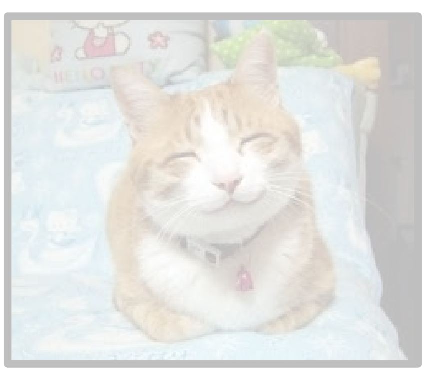

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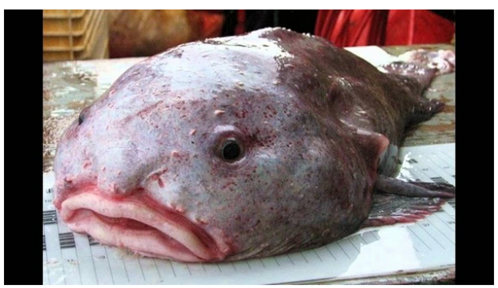

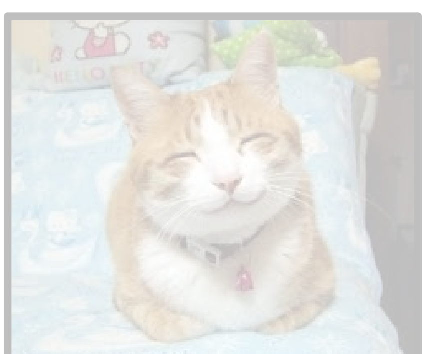

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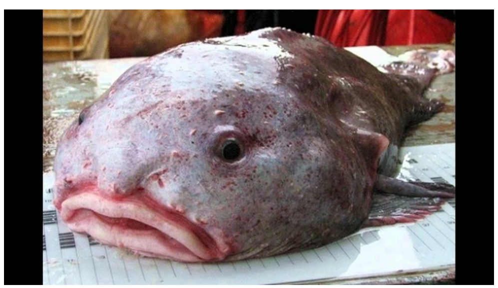

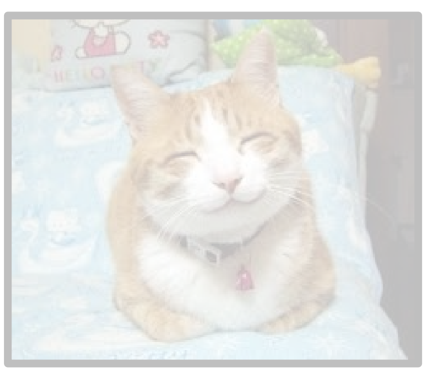

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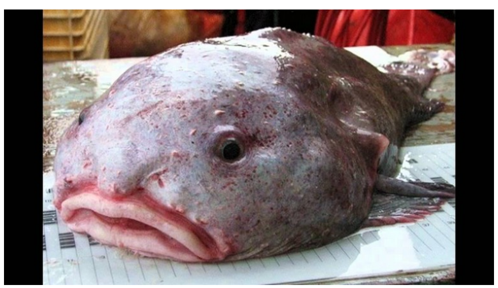

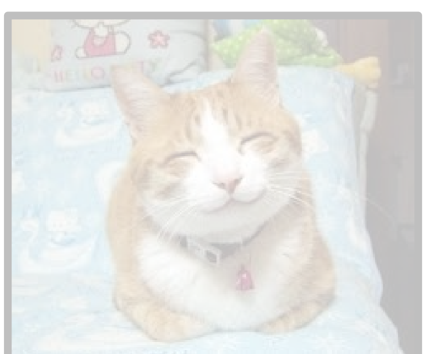

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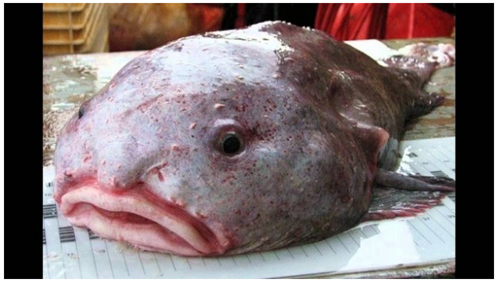

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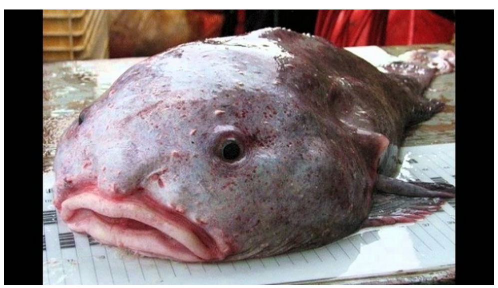

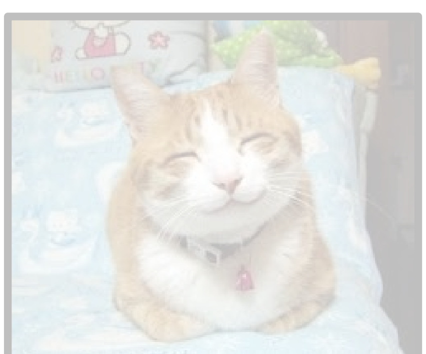

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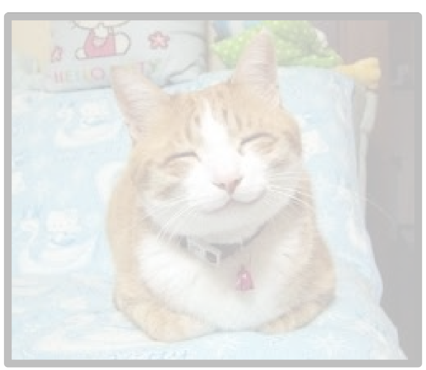

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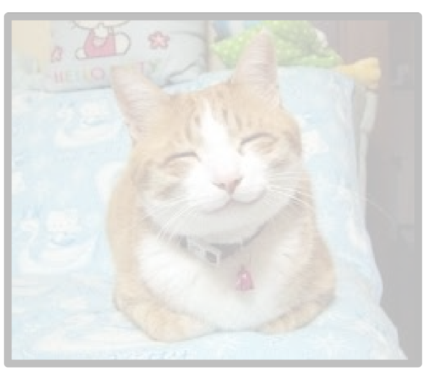

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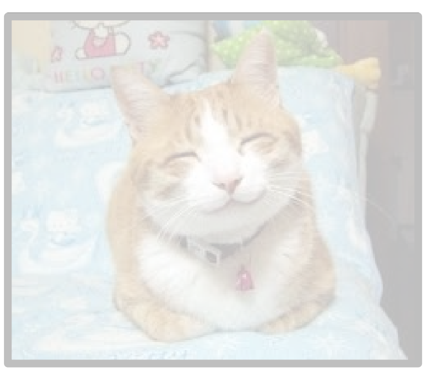

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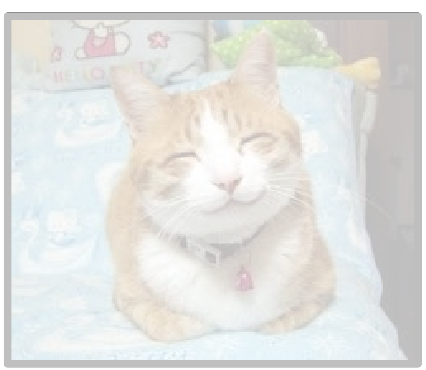
$\exists x.$ (~~*Blobfish*(x) ∧ *Cute*(x)~~)



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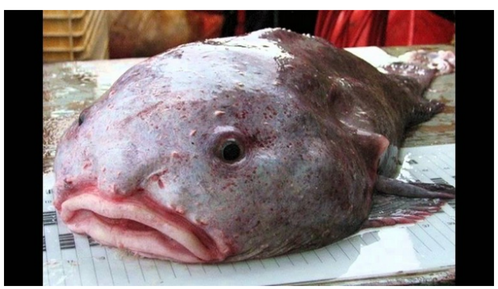

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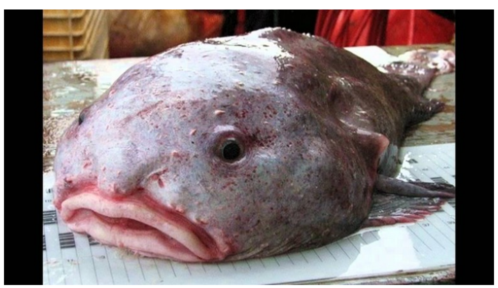

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“Some P is a Q”

translates as

$$\exists x.\ (P(x) \land Q(x))$$

Useful Intuition:

Existentially-quantified statements are false unless there's a positive example.

$$\exists x.\ (P(x) \land Q(x))$$

If x is an example, it must have property P on top of property Q.

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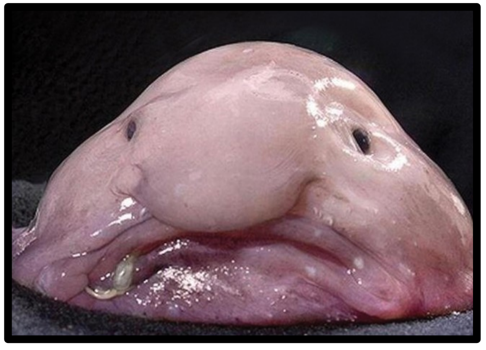

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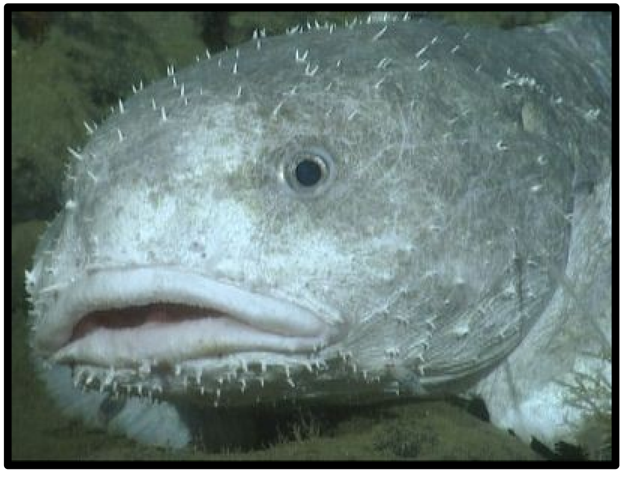

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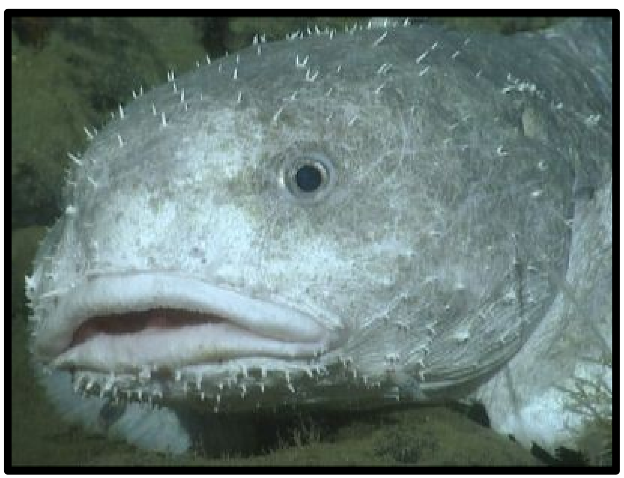


Slight aside: blobfish actually look totally normal underwater!

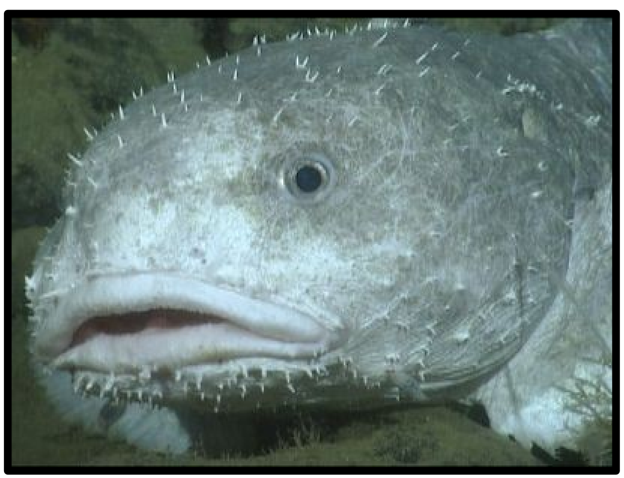
A Correct Translation

Some blobfish is cute.

$\exists x. (\textit{Blobfish}(x) \land \textit{Cute}(x))$

???

Your call :)



Slight aside: blobfish actually look totally normal underwater!

Good Pairings

- The ∀ quantifier *usually* is paired with →.

$$\forall x.\ (P(x) \to Q(x))$$

- The ∃ quantifier *usually* is paired with ∧.

$$\exists x.\ (P(x) \land Q(x))$$

- In the case of ∀, the → connective prevents the statement from being *false* when speaking about some object you don't care about.
- In the case of ∃, the ∧ connective prevents the statement from being *true* when speaking about some object you don't care about.

The Aristotelian Forms

"All *A*s are *B*s"

$\forall x.\ (A(x) \to B(x))$

"Some *A*s are *B*s"

$\exists x.\ (A(x) \land B(x))$

"No *A*s are *B*s"

$\forall x.\ (A(x) \to \neg B(x))$

"Some *A*s aren't *B*s"

$\exists x.\ (A(x) \land \neg B(x))$

It is worth committing these patterns to memory. We'll be using them throughout the day and they form the backbone of many first-order logic translations.

Let’s take a five minute break!

What we’ve covered:

- Set theory
  - Element of, subset of
  - Combining sets (union, intersection, etc.)
  - Power set
  - Cardinality
- Mathematical proofs
  - Direct proofs
  - Indirect proofs

Why?

- Set theory is a language we can use to pin down abstract concepts
- Largely, discrete math is a set of tools to help us answer really interesting questions
- Broadly applicable approach to problem solving

Let’s do a proof together!

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

What We're Assuming	*What We Need To Show*

When confronted with a theorem to prove, the first step is to make sure you understand where you're starting and where you're going.

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

What We're Assuming	*What We Need To Show*
• A and B are sets.	• $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

What We're Assuming

- A and B are sets.

What We Need To Show

- $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

Let's unpack this!

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

What We're Assuming	*What We Need To Show*
• A and B are sets.	• $\wp(A) \cap \wp(B) = \wp(A \cap B)$. Let's unpack this!

Talk with your neighbors and figure out:

- What is the definition of $\wp(S)$?
- What is the definition of $S \cap T$?
- How do you show two sets are equal to one another?

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

What We're Assuming

- A and B are sets.

What We Need To Show

- $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that $S = T$, show that $S \subseteq T$ and $T \subseteq S$

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A great proofwriting strategy is to ***write down relevant definitions***. This gives you a better sense of what you need to prove and what tools you have at hand.

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

What We're Assuming

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How can we apply this general template to our specific problem?

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

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How can we apply this general template to our specific problem?

S and T are *placeholder variables* here – what is S and what is T?

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

What We're Assuming

- A and B are sets.

What We Need To Show

- $\wp(A) \cap \wp(B) = \wp(A \cap B)$.
  - $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.
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What We Need To Show

- $\wp(A) \cap \wp(B) = \wp(A \cap B)$.
  - $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.
  - $\wp(A \cap B) \subseteq \wp(A) \cap \wp(B)$.

How do we show that one set is a subset of another?

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

What We're Assuming

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What We Need To Show

- $\wp(A) \cap \wp(B) = \wp(A \cap B)$.
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  - $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.
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How can we apply this general template to our specific problem?

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
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Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

What We're Assuming

- A and B are sets.
- $S \in \wp(A) \cap \wp(B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that $S = T$, show that $S \subseteq T$ and $T \subseteq S$
- In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that $x \in T$

What We Need To Show

- $\wp(A) \cap \wp(B) = \wp(A \cap B)$.
  - $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.
    - $S \in \wp(A \cap B)$.
  - $\wp(A \cap B) \subseteq \wp(A) \cap \wp(B)$.

Theorem: If A and B are sets, then ℘(A) ∩ ℘(B) = ℘(A ∩ B).

What We're Assuming

- A and B are sets.
- S ∈ ℘(A) ∩ ℘(B).

Relevant Definitions

- ℘(S) = { T | T ⊆ S }
- For all x in S ∩ T, x ∈
- In general to show tha
  show that S ⊆ T and T
- In general to show tha
  pick an arbitrary x ∈ S
  x ∈ T

What We Need To Show

- ℘(A) ∩ ℘(B) = ℘(A∩B).
  - ℘(A) ∩ ℘(B) ⊆ ℘(A∩B).
    - S ∈ ℘(A∩B).

Notice how we took the theorem we're trying to prove and unpacked it into simpler statements. Let's go and try to prove ℘(A) ∩ ℘(B) ⊆ ℘(A∩B) using the starting and ending points we identified here.

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.

- $S \in \wp(A \cap B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that $S = T$, show that $S \subseteq T$ and $T \subseteq S$
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- In general to show that $S = T$, show that $S \subseteq T$ and $T \subseteq S$
- In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that

Right now, we have no idea how to get from the start to our goal. It can be helpful at this point to just start writing down anything we know and apply any relevant definitions.

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.

What do we know about S based on this?

- $S \in \wp(A \cap B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that $S = T$, show that $S \subseteq T$ and $T \subseteq S$
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- Pick $S \in \wp(A) \cap \wp(B)$.
- $S \in \wp(A)$ and $S \in \wp(B)$.
- $S \in \wp(A \cap B)$.

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- Pick $S \in \wp(A) \cap \wp(B)$.
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- $\wp(S) = \{ T \mid T \subseteq S \}$
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Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.
- $S \in \wp(A)$ and $S \in \wp(B)$.
- $S \in \wp(A \cap B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that $S = T$, show that $S \subseteq T$ and $T \subseteq S$
- In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that

Another strategy we can try is to **work backwards**. That is, given where we want to end up, what would we have to show first in order to make that conclusion?

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.
- $S \in \wp(A)$ and $S \in \wp(B)$.

What would we have to show in order to conclude that $S \in \wp(A \cap B)$?

- $S \in \wp(A \cap B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that $S = T$, show that $S \subseteq T$ and $T \subseteq S$
- …neral to show that $S \subseteq T$, …n arbitrary $x \in S$, show that

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Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.
- $S \in \wp(A)$ and $S \in \wp(B)$.

What would we have to show in order to conclude that $S \in \wp(A \cap B)$?

- $S \subseteq A \cap B$.
- $S \in \wp(A \cap B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that S = T, show that $S \subseteq T$ and $T \subseteq S$
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Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.
- $S \in \wp(A)$ and $S \in \wp(B)$.

- $S \subseteq A \cap B$.
- $S \in \wp(A \cap B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that $S = T$, show that $S \subseteq T$ and $T \subseteq S$
- In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that $x \in T$

Take a few minutes and try to fill in the rest of these steps. Think about:

- How do you show a set is a subset of another?
- What can you say about S relative to A and B?

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.
- $S \in \wp(A)$ and $S \in \wp(B)$.
- $S \subseteq A$ and $S \subseteq B$.

  - $S \subseteq A \cap B$.
- $S \in \wp(A \cap B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that $S = T$, show that $S \subseteq T$ and $T \subseteq S$
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Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.
- $S \in \wp(A)$ and $S \in \wp(B)$.
- $S \subseteq A$ and $S \subseteq B$.

- $S \subseteq A \cap B$.
- $S \in \wp(A \cap B)$.

We want to show that $S \subseteq A \cap B$ so we'll pick an $x \in S$ and show that $x \in A \cap B$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that S = T, show that $S \subseteq T$ and $T \subseteq S$
- In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that $x \in T$

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.
- $S \in \wp(A)$ and $S \in \wp(B)$.
- $S \subseteq A$ and $S \subseteq B$.
  - Pick $x \in S$ so $x \in A$.
  - $x \in S$ so $x \in B$.
  - $S \subseteq A \cap B$.
- $S \in \wp(A \cap B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
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Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.
- $S \in \wp(A)$ and $S \in \wp(B)$.
- $S \subseteq A$ and $S \subseteq B$.
  - Pick $x \in S$ so $x \in A$.
  - $x \in S$ so $x \in B$.
  - $x \in A \cap B$.
  - $S \subseteq A \cap B$.
- $S \in \wp(A \cap B)$.

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Rough Outline

- Pick $S \in \wp(A) \cap \wp(B)$.
- $S \in \wp(A)$ and $S \in \wp(B)$.
- $S \subseteq A$ and $S \subseteq B$.
    - Pick $x \in S$ so $x \in A$.
    - $x \in S$ so $x \in B$.
    - $x \in A \cap B$.
    - $S \subseteq A \cap B$.
- $S \in \wp(A \cap B)$.

Take a few minutes and write up a proof for $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$ using this outline.

Then swap proofs with a neighbor and critique each other!

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Proof: Let A and B be sets. We need to show that $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Consider an arbitrary $S \in \wp(A) \cap \wp(B)$. This means that $S \in \wp(A)$ and $S \in \wp(B)$, so $S \subseteq A$ and $S \subseteq B$. We need to prove that $S \in \wp(A \cap B)$, meaning that we need to prove that $S \subseteq A \cap B$.

To prove that $S \subseteq A \cap B$, consider any $x \in S$. Since $x \in S$ and $S \subseteq A$, we know that $x \in A$. Similarly, since $x \in S$ and $S \subseteq B$, we know that $x \in B$. Collectively, we've shown that $x \in A$ and $x \in B$, so we see that $x \in A \cap B$. This means that every $x \in S$ satisfies $x \in A \cap B$, so $S \subseteq A \cap B$, which is what we needed to show. ■

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Proof: Let A and B be sets. We need to show that
$\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Are you clearly stating what you're assuming and what you're trying to prove?

To prove that $S \subseteq A \cap B$, consider any $x \in S$. Since $x \in S$ and $S \subseteq A$, we know that $x \in A$. Similarly, since $x \in S$ and $S \subseteq B$, we know that $x \in B$. Collectively, we've shown that $x \in A$ and $x \in B$, so we see that $x \in A \cap B$. This means that every $x \in S$ satisfies $x \in A \cap B$, so $S \subseteq A \cap B$, which is what we needed to show. ■

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

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Are you making specific claims about specific variables? Your proof should NOT have statements of the form "every element of S" or "every subset of S".

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Proof: Let ***A*** and ***B*** be sets. We need to show that
$\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Consider an arbitrary ***S*** $\in \wp(A) \cap \wp(B)$. This means that $S \in \wp(A)$ and $S \in \wp(B)$, so $S \subseteq A$ and $S \subseteq B$. We need to prove that $S \in \wp(A \cap B)$, meaning that we need to prove that $S \subseteq A \cap B$.

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Are all variables properly introduced and scoped? You should be able to point at every variable and say that it is either:

1) an arbitrarily chosen value
2) an existentially instantiated value
3) an explicitly chosen value

t

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Proof: Let A and B be sets. We need to show that $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

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To pro ... $\in S$
and S ... and
$S \subseteq B$, ... that
$x \in A$ a ... that
every ... what
we nee

Are you applying definitions appropriately?

Example: $S \in \wp(A)$ and the power set is the set of all subsets so $S \subseteq A$.

Lemma: If A and B are sets, then $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

Proof: Let A and B be sets. We need to show that
$\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

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To pro … S
and S … and
$S \subseteq B$, … that
$x \in A$ a… that
every … what
we nee…

Are you applying definitions appropriately?

Example: $S \in \wp(A)$ ~~and the power set is the set of all subsets~~ so $S \subseteq A$.

Proofwriting Strategies

- ***Articulate a Clear Start and End Point***
  - What are you assuming? What are you trying to prove?
- ***Write Down Relevant Terms and Definitions***
  - Identify existing tools to help you get from your starting point to your ending point
- ***Work Backwards***
  - Use your end goal to figure out intermediate steps

Theorem: If A and B are sets, then $\wp(A) \cap \wp(B) = \wp(A \cap B)$.

What We're Assuming

- A and B are sets.

What We Need To Show

- $\wp(A) \cap \wp(B) = \wp(A \cap B)$.
  - $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.
  - $\wp(A \cap B) \subseteq \wp(A) \cap \wp(B)$.

Relevant Definitions

- $\wp(S) = \{ T \mid T \subseteq S \}$
- For all x in $S \cap T$, $x \in S$ and $x \in T$
- In general to show that $S = T$, show that $S \subseteq T$ and $T \subseteq S$
- In general to show that $S \subseteq T$, pick an arbitrary $x \in S$, show that $x \in T$

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Good exercise: Try doing the other half of this proof!

Next Time

- ***First-Order Translations***
  - How do we translate from English into first-order logic?
- ***Quantifier Orderings***
  - How do you select the order of quantifiers in first-order logic formulas?
- ***Negating Formulas***
  - How do you mechanically determine the negation of a first-order formula?
- ***Expressing Uniqueness***
  - How do we say there’s just one object of a certain type?